SURFACE MODELLING FOR DIES

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

by
DEBADUTTA MISHRA

to the
DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR
FEBRUARY, 1992

CERTIFICATE

21/2/92

It is certified that the work contained in the thesis entitled " SURFACE MODELLING FOR DIES ", by Mr. Debadutta Mishra (Roll No. 9010514) has been carried out under our supervision and the work has not been submitted elsewhere for a degree.

Prof. J.L. Batra
Dept. of Industrial
and Management Engg.
I.I.T., KANPUR .

Prof. B. Sahay
Dept. of Mechanical
Engg.
I.I.T. , KANPUR .

February, 1992

ACKNOWLEDGEMENT

I take this opportunity to express my deep sense of gratitude to Prof. B. Sahay and Prof. J. L. Batra for the guidance, suggestions and encouragement they have provided during the course of this work.

I am extremely grateful to the coordinator and the staff of the CAD Project for providing me all sort of facilities available in their laboratory in support of this thesis work.

I would like to thank all my friends who have made my stay at IITk a memorable one. Special thanks to Mr. Ashok Naik for all the help he has extended in academic as well as non-academic matters.

Debadutta Mishra

CONTENTS

ABSTRACT

The work attempts to study and implement die sculptured surfaces on computer using various methods available in literature. The choice of parametric equation, particularly B-spline system is justified for the surface modelling . 3D die surfaces have been generated by defining control points and by data fitting technique. Most of the required die sculptured surfaces are being generated using sweep technique of 2D curves. Open surfaces are modelled by sweeping 2D curves on two boundary curves to meet the requirement of forging or die cast surfaces. Extrusion die surfaces are obtained by cylindrical sweep technique considering the minimum requirement of the extrusion ratio to be constant. The joining of surface patches with C^1 and C^2 continuity is implemented to improve the efficiency of surface design.

The system is based on the mathematical models suggested by various researchers engaged either in the field of surface modelling or in development of integrated CAD/CAM of dies. The software is implemented on HP-9000 work station using Starbase graphics and C language.

LIST OF FIGURES

NOMENCLATURE

a	surface name
$B_{i,n}$, $B_{j,m}$	Bernstein Polynomials in Bezier Curve
$b_0(v)$, $b_1(v)$	Position Vector of Boundary Curves
c	Connecting Surface
$c(u)$	Position Vector of Curve c
$\dot{c}(u)$	First Derivative of Curve $c(u)$
$\dot{c}'(u)$	Second Derivative of Curve $c(u)$
Dx, Dy, Dz	Coordinates of Data Points
[D]	Matrix Form of Data
$E_{i,j}$	Control Points Obtained From Data $e_{i,j}$
$e_{i,j}$	Data Points on a Curve $r_j(v)$
$f(v)$	Correction Factor
G	Coordinate of Guide Point
$g(v)$	Position Vector of Guide Curve
i, j	Index
k, ku, kv	Order of B-spline Curve
l	Segment Length of Extrusion Product
N_j	Number of Data for jth Curve
nu, nv	umber of Control Points Found for Fitting Curves and Surfaces
[p]	Matrix Form of Position Vector
$P(u)$	Parametric Position Vector of Curve
$p(u,v)$	Biparametric Position Vector for Surface
$q(u,v)$	Biparametric Position Vector for Blended Surface
Ri	Any Positive Constant for Surface connection
r_j	Parametric Position for jth Curve

$s(u,v)$	Position Vector of Surface
$s_0(u), s_1(v)$	Position Vector of Section Curve
T, T_x, T_y, T_z	Tolerances
u	Parameter
v	Parameter
$x(u)$	x-coordinate for u Parameter
X_i	Knot Vector
X_c	X-coordinate of Centroid
$y(u)$	Y-coordinate of for u Parameter
y_c	Y-coordinate of Centroid
$z(u)$	Z-coordinate for u Parameter
$IV(v)$	Intersection Vector in x-y Plane
ϕ	Angle between -x axis and IV(v) Vector in Section Plane
$\theta(v)$	Angle between x-axis and IV(v) in x-y Plane
$\alpha(v)$	Blending Function
$B(v)$	Blending Function
ρ	Radius of Curvature

CHAPTER 1

INTRODUCTION

In the 1970s, the trend in advanced mass production technology was towards die based processing and moulding. Manufacturing using dies and moulds offer numerous advantage over other manufacturing methods such as machining and joining processes. In medium to large production, moulding and forming methods become superior in terms of lower unit costs. In addition, material utilization is very high and rejection rates become very low. Dies are used in a variety of industries including pressing, forging, casting , rubber, glass, ceramics and plastics. Manufactured goods using cast parts range from airplanes to toasters and from automobiles to toys.

Now a days, such industries as automobile, electrical goods and heavy machinery units use dies to manufacture a majority of their basic components. The highest level of technology is required for accurate and efficient die making, so that a wide variety of manufactured goods can be produced economically.

The higher cost of overheads in addition to customer demand for variety of products at lower price and quick delivery make automation of die making industry an important priority. The spectacular growth of CAD/CAM - Computer Aided Design and Computer Aided Manufacture, which emerged within a last couple of decades as a result of revolutionary technology advancement of micro and digital computer, has helped to increase Engineering Productivity

tremendously.

There are inherent difficulties in designing extrusion and forging dies for making discrete parts. Traditionally, die design is considered more of an art than science. The analysis of irregular shaped forged dies or of streamlined extrusion dies for complex non-axis symmetric sections has been limited in the past due to complexity and high cost of die design and manufacture. However, the advances in the use of computers in Engineering Design and Manufacture have opened up new avenues in such application.

In this context, we shall see later how the die surfaces can easily be represented on a computer screen before any analysis for modelling purpose which is done as the first step toward integrated CAD/CAM of dies.

1.1 LITERATURE SURVEY

Hajimu Kishi of Fujtitsu Fanuc Ltd[1] in 1970 defined and classified the sculptured surfaces as required for dies. He also suggested some technique to obtain die sculptured surfaces from sectional diagrams. Fanuc Ltd used this technique to develop a software FAPT Die-II for their CNC machines for design and manufacturing purposes . The software was designed for the desk top type CNC tape preparation system with graphic system considering the fact that expensive CAD/CAM systems are readily not adaptable to the economic need of an industry composed of small scale job shops.

Gunasekara and Hoshino [3] carried out many theoretical and experimental studies on extrusion dies having circular entry shape and regular polygonal exit shape and generated optimum die profiles using computers and used the data to produce engineering drawing and NC tapes for machining of EDM electrodes and subsequent electro discharge machining of dies.

Dunkan [4] has reviewed the research in sheet metal forming using computer aided design for limit strain applications. He has also described the application of this simple analysis to the problem of shape control in smoothly curved plane.

Gunasekara et al[5] developed an efficient software for analysis of metal flow during forging of complex shapes using finite element method. They also developed a concept for design

of streamlined dies of arbitrary product shape and manufactured dies for the extrusion of complex structures. Visualization and understanding of complex 3D geometry are enhanced by the use of shaded color graphics.

Gunasekara[5] developed a software named STREAM which consists of three packages for modelling and design of shaped extrusion dies including re-entrant section and computer aided manufacture of the die surface.

Nagpal[5] used a quadratic polynomial function to design the streamlined extrusion die for complicated section.

Sabaroff[6] used computer technique for generating the spiral bevel tooth surface geometry by simulating gear cutting process and manufactured EDM electrodes for the forged dies.

Yang[7] proposes the conformal transformation for extrusion of helical shapes. However, the model offers limited extrusion shapes.

Shev and Lee[7] developed an integrated CAD/CAM/CAE method for optimum die surface design of 3D extrusion surfaces. Tension parameter, geometric coordinates and polynomial curves are used to define a biparametric surface for the dies. The optimum die surface subsequently was manufactured by them using CAD/CAM software and NC machining. The proposed model claimed to obtain better extrusion performance.

1.2 SURFACE REPRESENTATION SCHEMES

Though the representation and design of surfaces on computer is still a state of art subject, numerous ways are suggested for the correct representation. As such there is no universal form to define all real surfaces, hence the surface appropriate for a given problem depends on the application. Here are few examples of surface representation schemes carried on by different researchers.

Polynomial cubic patches, Bezier patches, B-spline patches, Ferguson patches [8] are few surfaces patches capable of proving useful surface geometries.

The Math Group at University of Utah[11] devised a robust surface generation method applicable to arbitrarily located data using interpolation technique.

Vergeest et al at Delft University of Technology[12] have suggested carrying out a software named SIPSURF for designing arbitrary surfaces and subsequent machining of these surfaces.

Choi [15] has described a method of constructing a smooth sculptured surface by using sweep technique. The software DUCT is partly based on this technique.

Peigle [16] in his survey ON NURBS has described how a rational B-spline can be used as a powerful tool for any type of surface generation.

There are many software developed for surface modelling by various industries like Boeing, SDRC, Intergraph corporation etc These softwares cater to specific needs of the industry. The development of surface model is still under research and the best modeller coping with all the problems associated in a surface modeller is yet to come[15].

1.3 OBJECTIVE OF THE PRESENT WORK

The objective of the present work is limited to surface modelling for forge dies (open surface)and extrusion dies(cylindrical surface). The surface design on the computer screen shows the final shape of the product. After reviewing literature available in CAD/CAM of dies, it is observed that most of the work are meant for specific problem like extrusion dies or forging of helical gears etc with analysis of material flow and load required for the operation.

In this work, an attempt has been made to model surfaces for dies as prescribed by Kishi[2] which can be assumed to be the first step of CAD for dies. A complete CAD of die design would have been as shown in the flow chart(Fig1.1). However, taking into account the volume of work involved in the analysis, it is emphasized in the modelling of various surfaces only and their presentation rather than detail analysis. Fig1.2 briefly describes our work.

We consider parametric curves and surfaces as two different tool to describe die sculptured surface. A basic surface can be formed either by supplying control points or by exact coordinates on the surface which could be fitted to a

approximated surface.

Sweep technique is used to describe most of the surfaces as given by Kishi [2]. Similarly area mapping technique[7] is used along with sweep to define cylindrical surface for extrusion dies.

An attempt is made to connect arbitrary shape surfaces with tangency or curvature continuity to facilitate the designer to describe blending etc.

The software is implemented on the HP-9000 work station using starbase graphics and C language in unix environment.

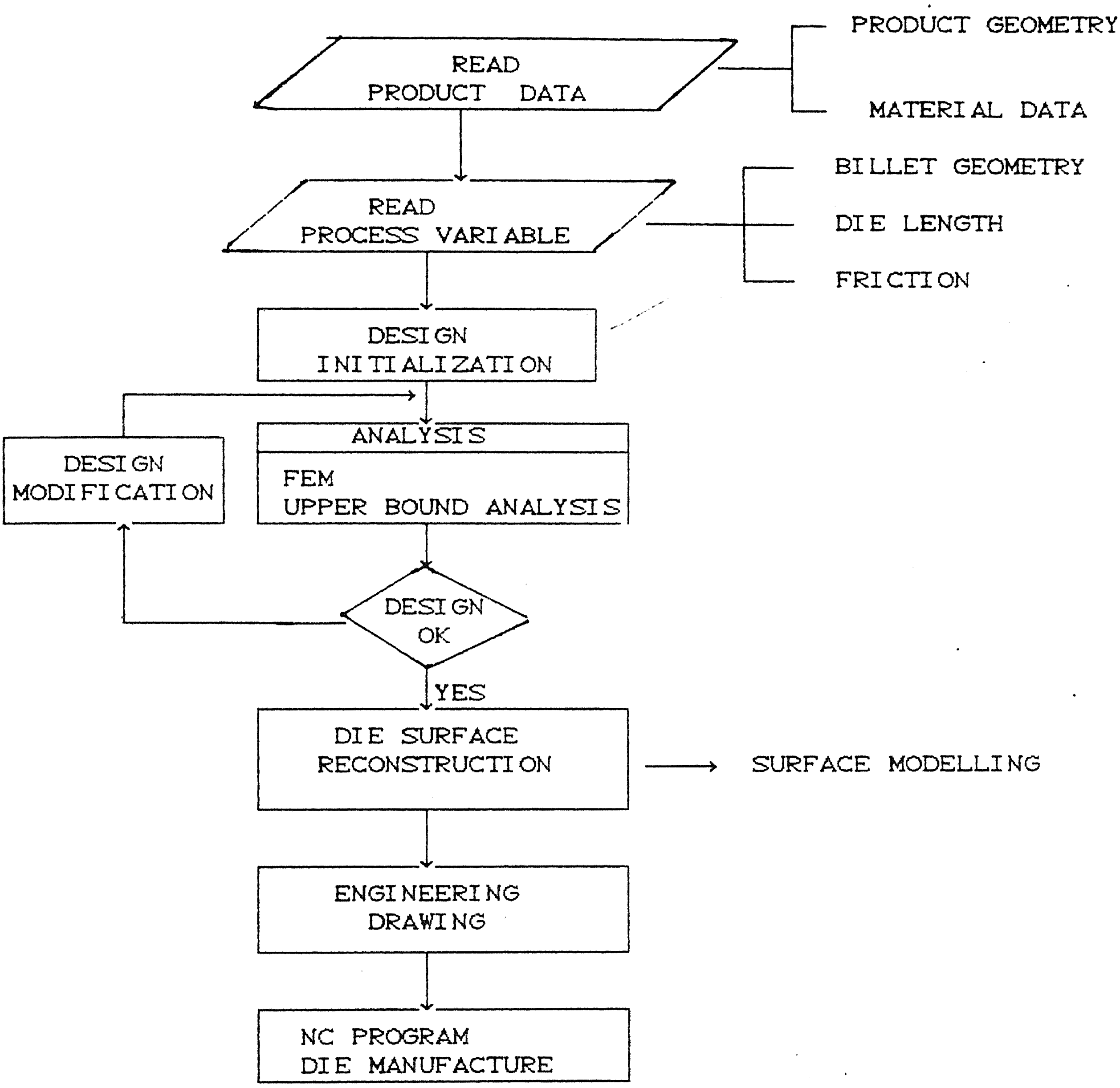


Fig. 1.1 FLOW CHART FOR DIE DESIGN AND MANUFACTURE

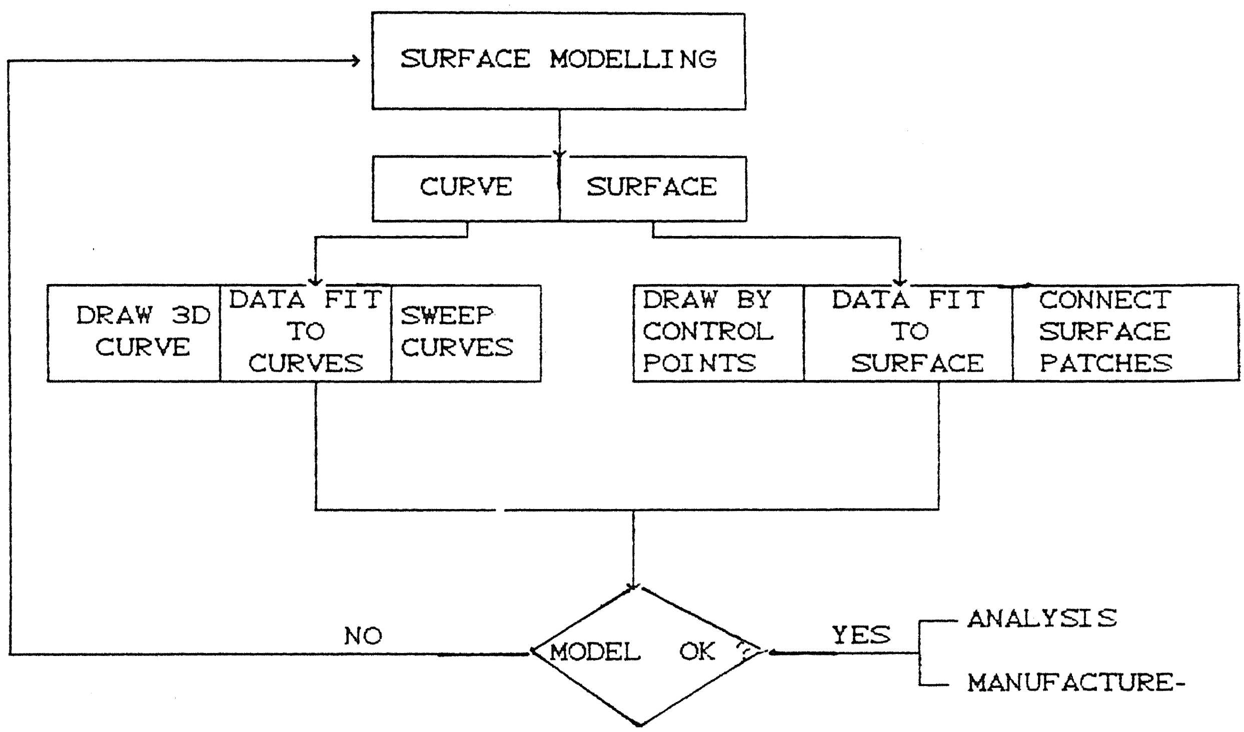

SURFACE MODELLING
CURVE
SURFACE
DRAW 3D CURVE
DATA FIT TO CURVES
SWEEP CURVES
DRAW BY CONTROL POINTS
DATA FIT TO SURFACE
CONNECT SURFACE PATCHES
NO
MODEL OK ?
YES
ANALYSIS
MANUFACTURE-

Fig. 1.2

CHAPTER2

DIE SCULPTURED SURFACE AND MATHEMATICAL FORMS

2.1 INTRODUCTION

Die shaping can be divided into two general types: 2 dimensional and 3 dimensional. These shapes can be characterized by planes, spheres, cones or cylinders etc so that NC milling machines or wire cut Electric Discharge machines can be used to machine these arithmetic shapes.

Typically the die makers receive specification from the industry in the form of sectional diagrams for 3D sculptured surfaces. With these specifications the die makers are expected to form the intermediate sections and finished shape so that it is functional or pleasing to the eye.

2.2 CLASSIFICATION AND DEFINITION OF DIE SCULPTURED SURFACE

The die sculptured surfaces are defined by a combination of two kinds of section curves. One is called basic curve and the other is called drive curve. The basic curve is a shape element which governs the motion of drive curve. The drive curve is a shape element which moves with reference to the basic curve. Fig 2.1 gives some example of die sculptured surfaces according to whether the condition in which the drive curve moves is parallel, radiate or normal[2]. Fig 2.2 shows sculptured surface for 2 drive curves. Fig 2.3 shows some classification of surfaces

depending on how the drive curve moves on the boundary curve.

2.3 CYLINDRICAL SCULPTURED SURFACE

Such shapes can be characterized by sectional curves on cylinders. The basic curve is fixed and as drive curve moves along the basic curve the drive curve shape changes from 1 to 2 Fig 2.4. The classification of cylindrical sculptured dies are shown in Fig 2.5 [2]. The extrusion die surfaces fall into this category.

2.4 MATHEMATICAL FORMS OF SCULPTURED SURFACES

The geometry of a die may be approximated by a collection of plane faced polyhedra with straight edges. However, such a representation may involve hundred or thousands of faces or edges and would clearly be cumbersome to generate and modify. Hence it is necessary to represent the surface of a die as a collection of curved sculptured surface patches bounded by curved edges. However, irrespective of the method used to represent complex surfaces the geometry has to be modelled in mathematical form suitable for a computer. The following are some properties that mathematical model should have[10] -

1. Control points - may be on the curve or outside on a polygon.
2. Multiple variable - a curve may have multiple values, i.e. for one particular coordinate(x,y or z), there may be multiple value of other coordinates.
3. Axis independence - the shape should be independent of reference axis chosen.

4. Global or Local control - a fine local adjustment to the surface should not affect the geometry as a whole.

5. Versatility - mathematical models of curves should be versatile

6. Orders of continuity - the designer may want to join pieces of curves or surfaces maintaining the continuity of the geometry defined by the same control of points, i.e. the models should have well defined geometric properties.

The above requirements have lead to the use of parametric or vector value methods for representation of curves and surfaces as given below.

For curves

$$P(u) = [\ X(u)\ Y(u)\ Z(u)\ ]. \tag{2.1}$$

For surfaces

$$S(u,v) = [\ X(u,v)\ Y(u,v)\ Z(u,v)]. \tag{2.2}$$

2.4 SURFACE PATCHES

Computer aided design objects are not solely concerned with pictorial properties of shape but with providing a geometric model or geometric data base which can then be used for design, analysis and manufacture. Some well-known surface patches used in various industries are bilinear surface, Coon's surface, Lofted surface, Bezier surface, B-spline surface and rational B-spline surface patches[9].

However, it is not easy for a designer to to use Coon's surface patches or Lofted surface patches to define any arbitrary shape. Bezier or B-spline systems are widely used for describing surfaces of automobile, ship hull, aeroplane body etc. In this

work we have used B-sline or Bezier curves for defining surface patches.

The parametric equation of bezier surface is -

$$P(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} P_{i,j} B_{i,n}(u)\ B_{j,m}(v) \qquad (2.3)$$

where the Bernstein Polynomials B's are given by

$$B_{i,n}(u) = {}^{n}C_{i}\ u^{i}\ (1-u)^{n-i} \qquad (2.4)$$

$$B_{j,m}(v) = {}^{m}C_{j}\ v^{j}\ (1-v)^{m-j} \qquad (2.5)$$

with $${}^{n}C_{i} = \frac{n!}{i!(n-i)!} \quad \text{and} \quad {}^{m}C_{i} = \frac{m!}{j!(m-j)!} \qquad (2.6)$$

$P_{i,j}$ are the vertices of the defining polygon net. n and m are one less than the total number of polygon vertices in the u and v direction respectively. u and v are two parameters.

The parametric equation for B-spline is -

$$S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} P_{i,j}\ N_{i,k}(u)\ \ M_{j,l}(v) \qquad (2.7)$$

where $N_{i,k}(u)$,$M_{j,l}(v)$ are B-spline basis functions in the biparametric u and v directions respectively.

$$N_{i,1} = \begin{cases} 1 & \text{if } X_i \leq u < X_{i+1}, \\ 0 & \text{else otherwise.} \end{cases} \qquad (2.8)$$

$$Ni,k = \frac{(u - X_i)\ N_{i,k}\ (u)}{X_{i+k-1} - X_i} + \frac{(X_{i+1} - u)\ N_{i+1,k}\ (u)}{X_{i+k} - X_{i+1}} \qquad (2.9)$$

and

$$M_{j.1} = \begin{cases} 1 & \text{if } Y_i \leq v < Y_{i+1}, \\ 0 & \text{else otherwise} \end{cases} \qquad (2.10)$$

$$M_{j,l} = \frac{(v-Y_j)\, M_{j,l}\ (v)}{Y_{j+l-1} - Y_j} + \frac{(Y_{j+1} - v)\, M_{j+1,l}\ (v)}{Y_{j+k} - Y_{i+1}} \qquad (2.11)$$

where $P_{i,j}$ are the vertices of defining polygon patches. For quadrilateral surface patches the defining polygon net must be topologically rectangular. The indices n and m are one less than total number of polygon points in u and v direction respectively. k and l are order of the curve in u and v direction. In the above equation 0/0 is taken as 0.

X_i and Y_i are called knot vectors which are important in deciding curve and surface shape. There are 3 types of knot vector namely - Open uniform, Close uniform and Non-uniform[8].

When open surface is required open uniform knot vector is used and given by -

$$\left.\begin{array}{ll} X_i = 0 & i < k\ , \\ X_i = i-k+1 & k \le i \le n\ , \\ X_i = n-k+2 & i > n\ . \end{array}\right\} \qquad (2.14)$$

The parameter u lies in the range of [0,n-k+2].

When a closed surface is required the close knot vector is used and given by -

$$X_i = i\ , \qquad (2.15)$$

and (k-2) polygon vertices have to be repeated at end/or beginning of each curve. The parameter lies in the range of (k-1) to n-k+1.

Nonuniform knot vector is useful in connecting separate

segments of a curve and defining quadratic curves like circle, ellipse etc and are decided by various mathematical forms[8].

Fig 2.6 and Fig 2.7 are two examples of open and close B-spline surfaces respectively.

NONUNIFORM RATIONAL B-SPLINE:

The rational form of B-spline offers a common mathematical equation for representing and designing both standard analytic shapes(conics,quadratics,surface of revolution etc) and free form surfaces precisely. The parametric model is given by-

$$S(u,v) = \frac{\sum_{i=o}^{n}\sum_{j=o}^{m} W_{i,j} P_{i,j} N_{i,k}(u) M_{j,l}(v)}{\sum_{i=o}^{n}\sum_{j=o}^{m} W_{i,j} N_{i,k}(u) M_{j,l}(v)} \qquad (2.16)$$

All notations are the same as given for a B-spline surface. $W_{i,j}$ is a weight function associated with each polygon control vertices. When W is 1 for all i,j rational B-spline reduces to ordinary B-spline[8].

2.5 WHY B-SPLINE IS BEST FOR DIE SURFACE ?

In the present work, B-spline is used as design curve or surface because it shows all necessary properties mentioned before. It has advantage over Bezier system as it allows local control which means the influence of any single vertex of polygon net is limited to only +k/2 to -k/2 span on each curve. Again when the number of defining polygon net vertices is equal to the

order of each curve B-spline curve reduces to Bezier curve.

In addition, it has some more advantages given below[15]-

1. It can be manipulated in number a of ways such as by tweaking polygon net vertices, order of the curve, using multiple polygon vertices,type of knots,multiple knots etc .
2. Evaluation is reasonably fast and computationally stable.
3. It has a powerful geometric tool kit;i.e. scaling, rotation, translation, parallel or perspective transformation can be carried out on the geometric database.
4. B-splines have clear geometric interpretations, making them particularly useful for designers who have a good knowledge of geometry especially descriptive geometry.
5. The B-spline surface algorithms is mostly supported by work stations.
6. The non-uniform rational form of B-spline(NURB) are genuine generalizations of nonrational and rational B-spline system as well as Bezier curves. But it needs more storage and demands careful choice of weights associated with each polygon vertices.

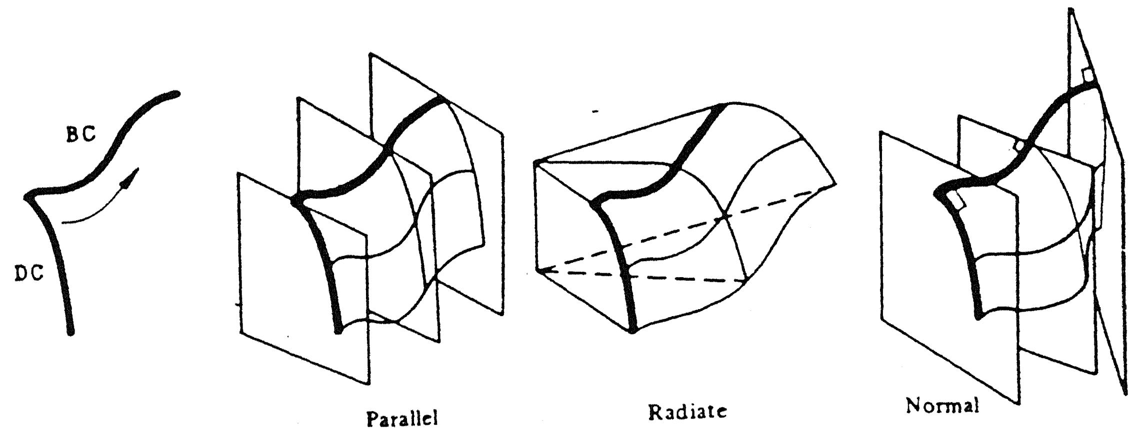


Fig. 2.1 Sculptured Surface Generated by Basic and Drive curves

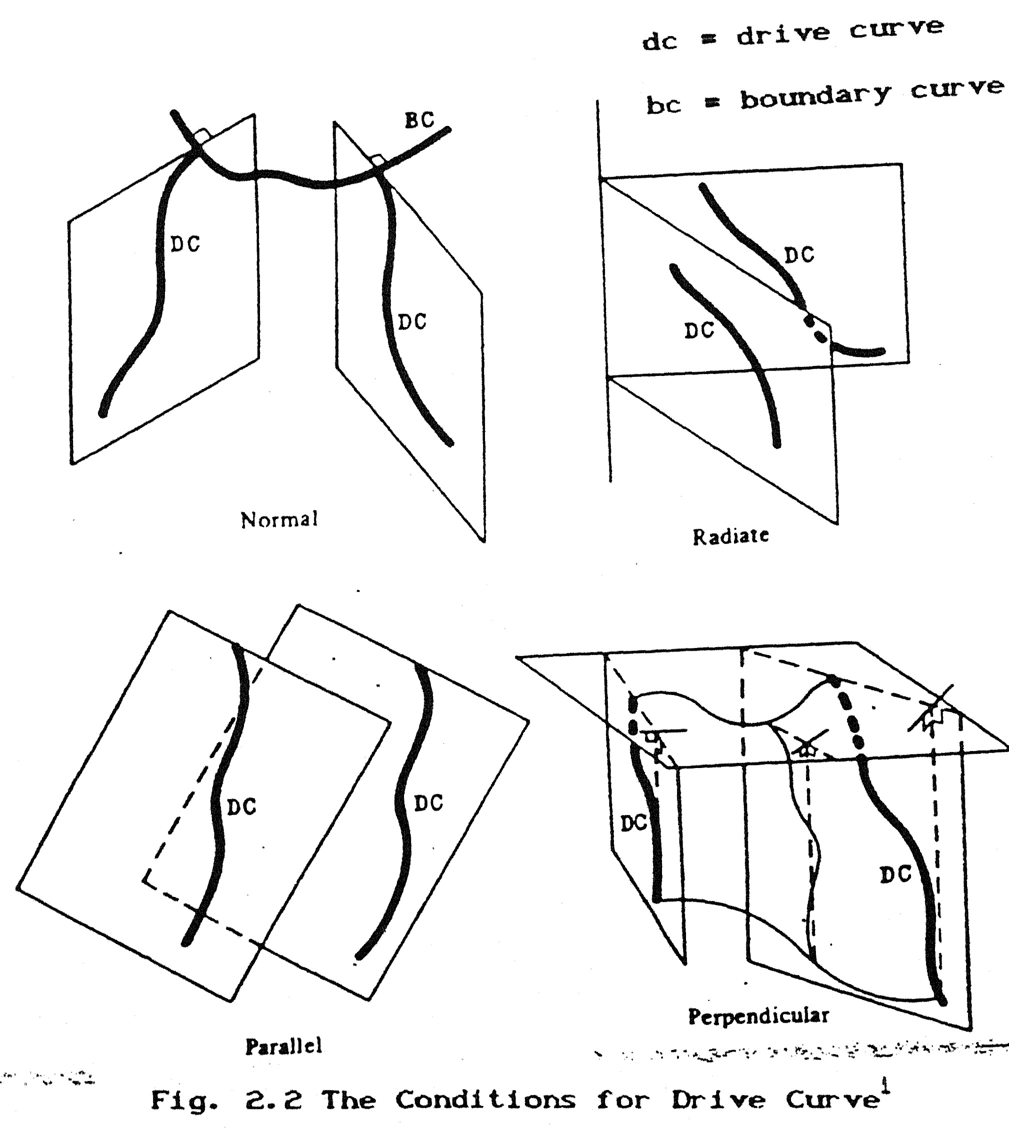


Fig. 2.2 The Conditions for Drive Curve[1]

sculptured surface digits : Ist digit = No. of bc, 2nd digit = No. of drive cur

3rd digit = condition in which dc moves on bc

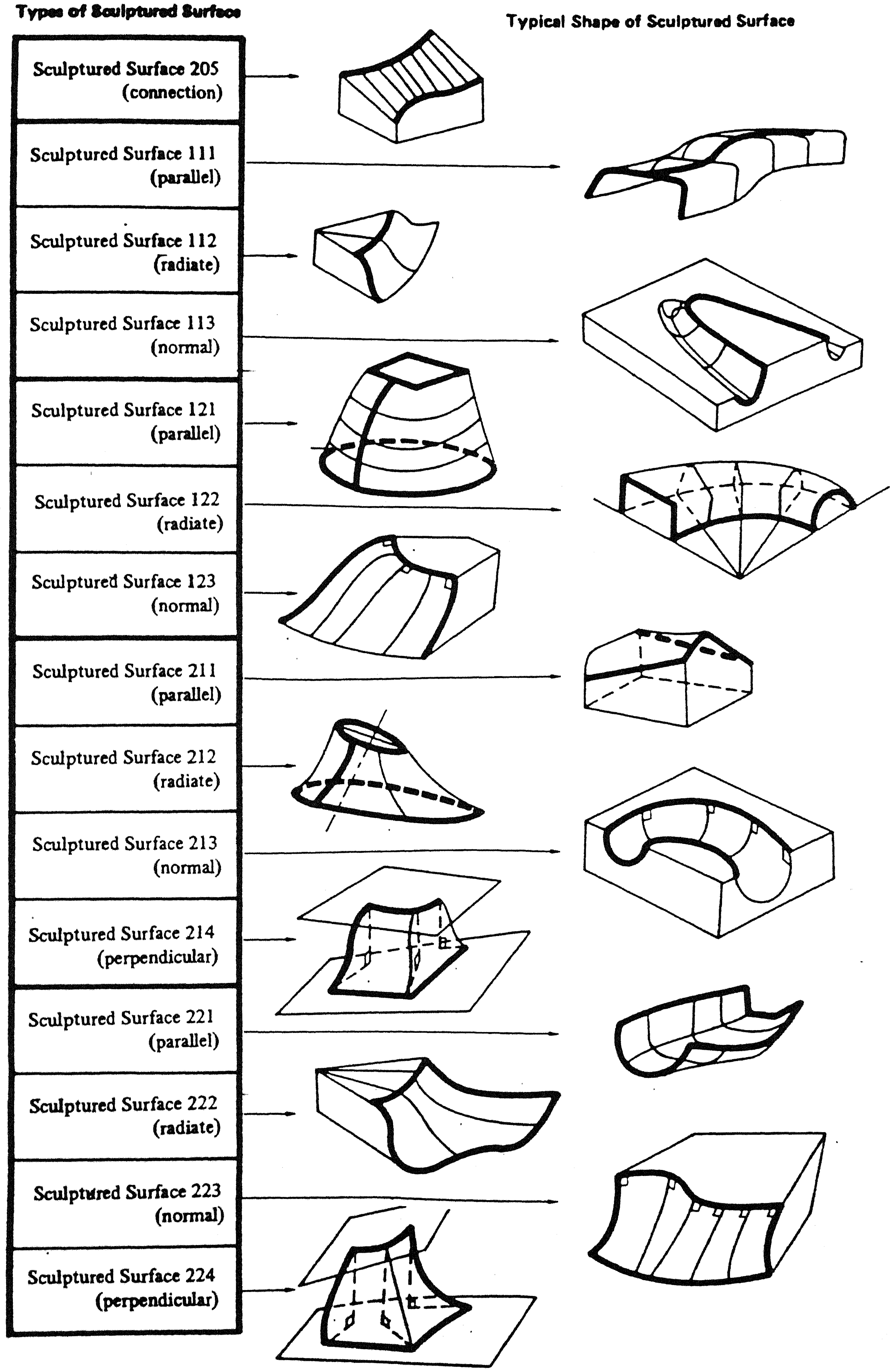


Fig. 2.3 Classification of Sculptured Surface[1]

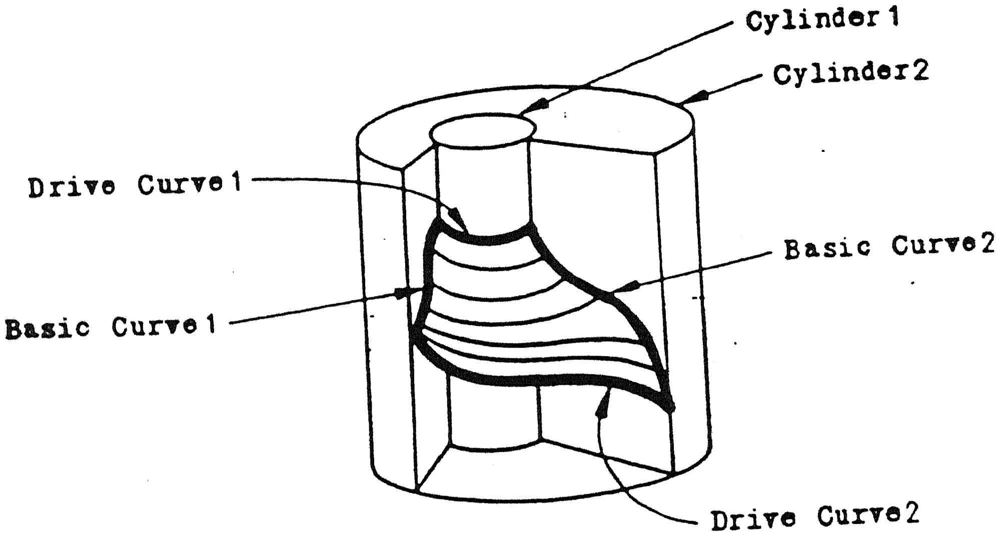


Fig. 2.4 Cylindrical Die Sculptured Surface[1]

Number of BCs	Number of DCs
0	2
1	1
1	2
2	1
2	2

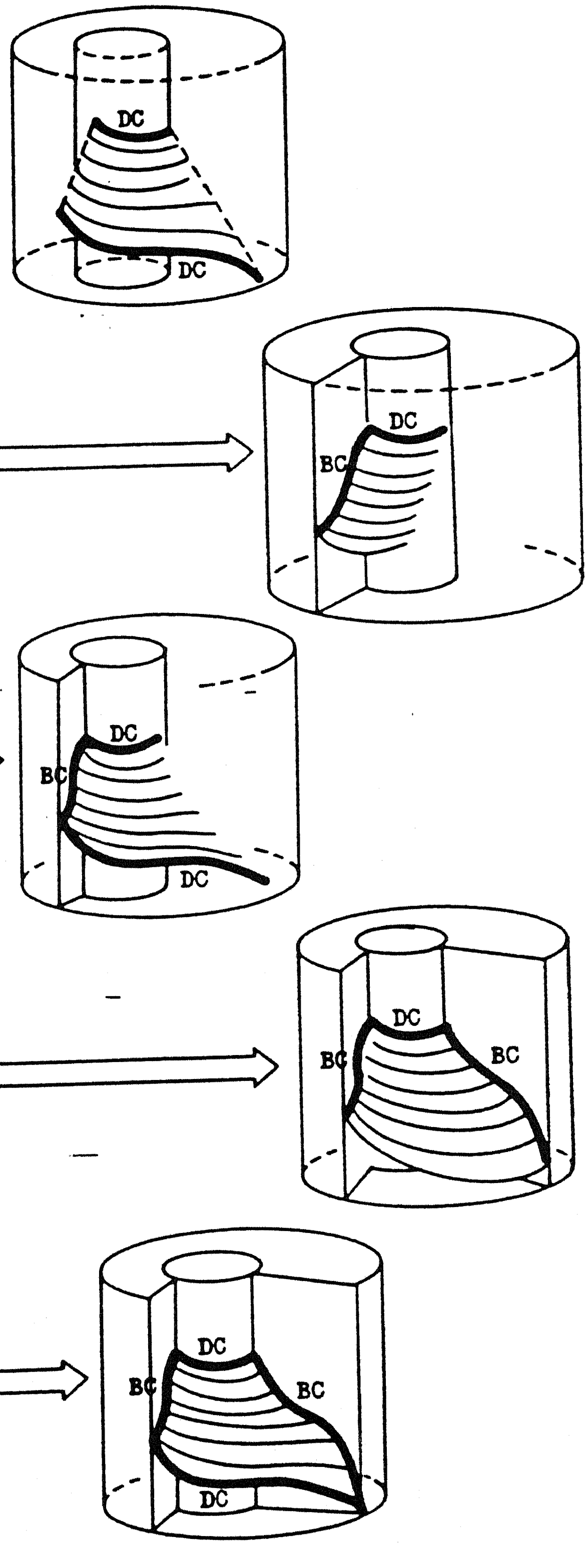


Fig. 2.5 Classification of Cylindrical Sculptured Surface[1]

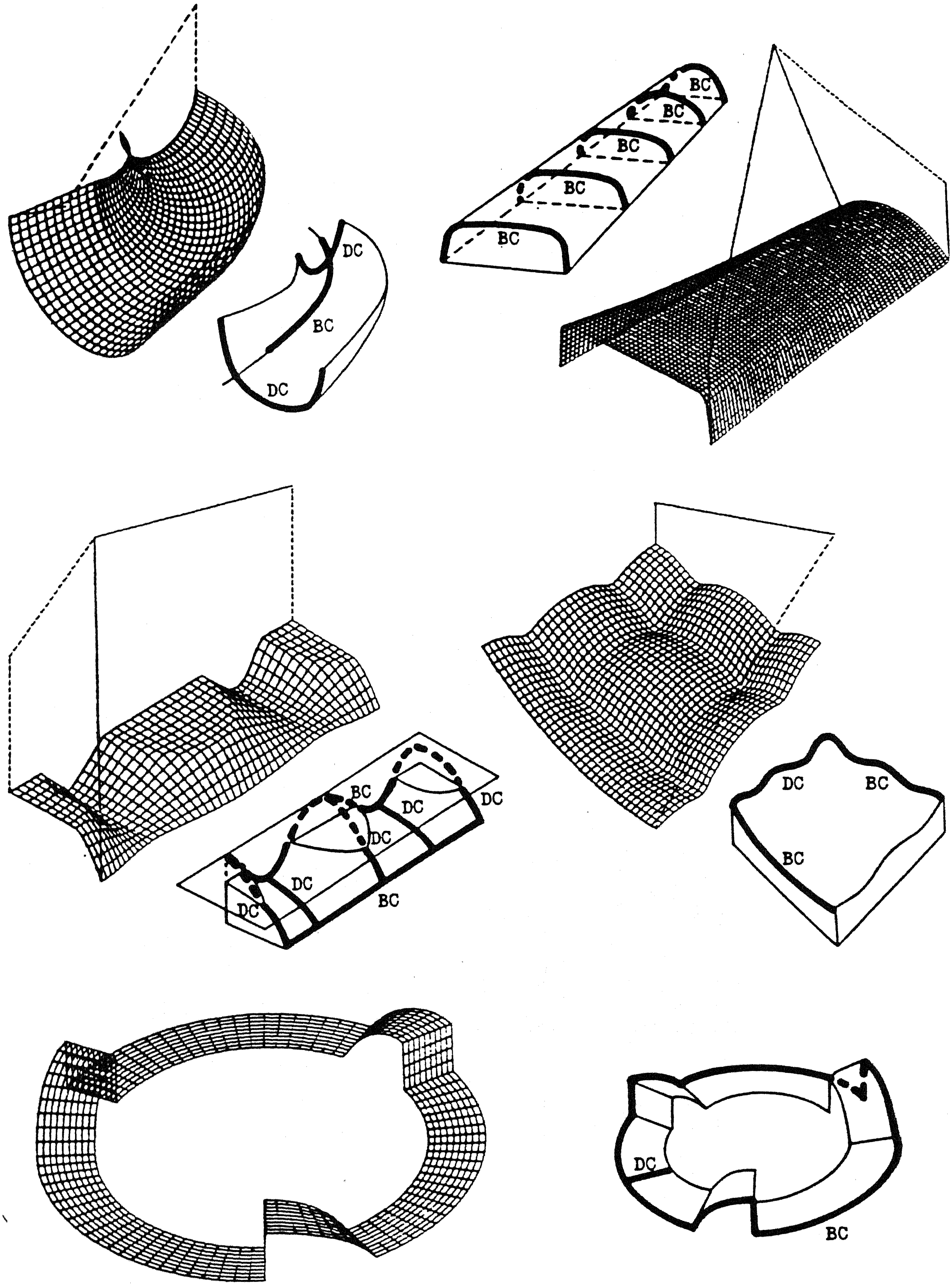


Fig. 2.6 Example of Die Sculptured Surface[1]

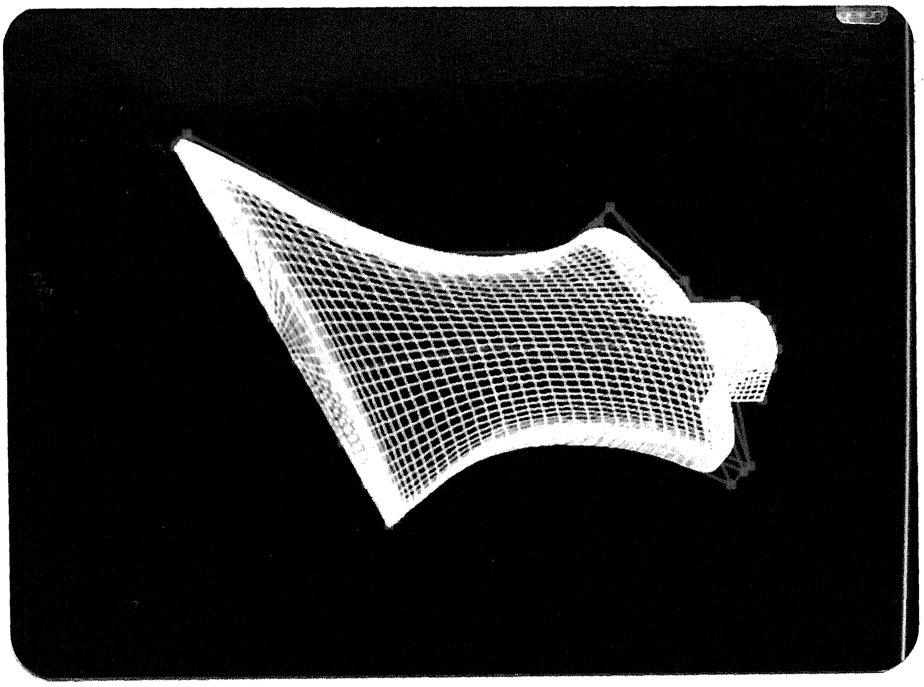


Fig. 2.7 Example of Open B-spline Surface

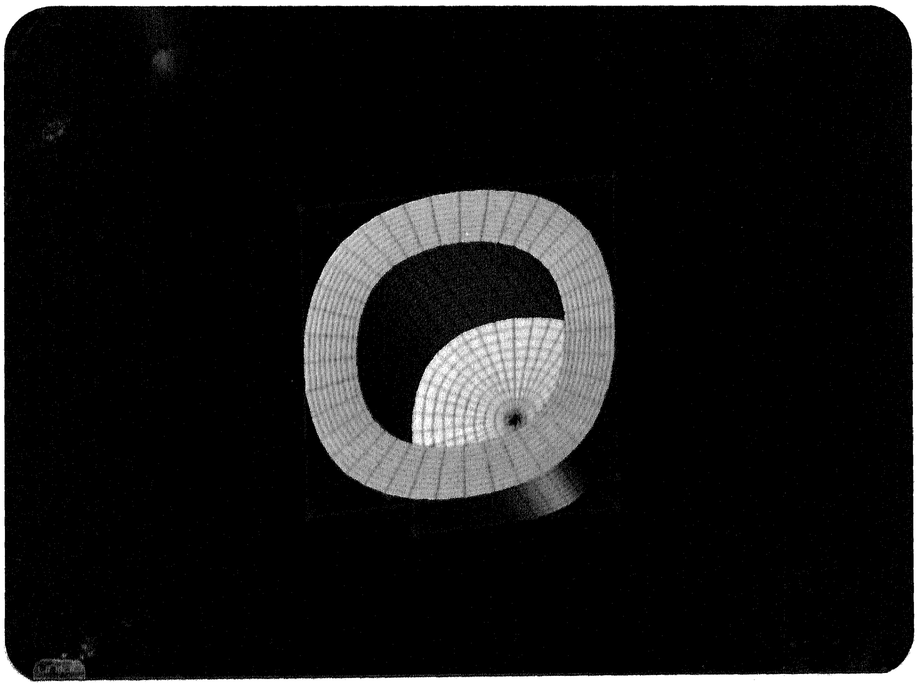


Fig. 2.8 Example of Close B-spline Surface

CHAPTER 3

SURFACE FITTING FOR SHAPE DESIGN

3.1 GENERAL INTRODUCTION

Surface fitting techniques have been developed in a wide variety and for a great number of different applications through interpolation or approximation scheme. The fitting methods usually vary with respect to the constraints which may be imposed such as boundary conditions, smoothness criteria or local deviation tolerance.

The interactive surface design can be carried out on Bezier/B-spline system doing operations like control points tweaking etc as described in Chapter 2. However, the designer has limited control over the actual surface by changing control polygon vertices, hence editing is not suitable for faring curve or surfaces. Again for B-spline surfaces, the tensor product concept hampers control point addition and deletion because a knot is associated with a row of de Boor control polygon(Basis function)[16].

Here a fitting of data point scheme is described which could be very useful for open or close die surface development. But the scheme is based on global method;i.e. change of any data point affect the surface as a whole however small may be. As a result the modelling is very fast for interactive process as long as the number of data is not excessively large.

The fitting method is implemented using B-spline system and the surface is optimized with respect to deviation from data

points in a least square sense which means the surface will deviate from any data point by less than some user specified distance.

3.2 B-SPLINE SYSTEM

The parametric form of B-spline surface is given in Chapter 2 Eq(2.8) to Eq(2.12). The equation can be written by 3 coordinate functions in two common real variables(parameters) u and v as

$$X(u) = \sum_{i=1}^{nu}\sum_{j=1}^{nv} Px_{i,j}\ N_{i,ku}(u)\ N_{j,kv}(v) \qquad (3.1)$$

$$Y(u) = \sum_{i=1}^{nu}\sum_{j=1}^{nv} Py_{i,j}\ N_{i,ku}(u)\ N_{j,kv}(v) \qquad (3.2)$$

$$Z(u) = \sum_{i=1}^{nu}\sum_{j=1}^{nv} Pz_{i,j}\ N_{i,ku}(u)\ N_{j,kv}(v) \qquad (3.3)$$

and $P_{i,j} = (\ Px_{i,j}\ Py_{i,j}\ Pz_{i,j}\ )$ (3.4)

are three dimensional control points in (nuxnv) size matrix. nu and nv are total number of control points. $N_{i,ku}$ and $N_{j,kv}$ are B-spline basis function given by Eq(2.9) and Eq(2.12). ku and kv are orders of the curve in u and v parameter direction.

The possible surface topology are :

1. Surface open in u and v direction (plane topology).
2. Surface open in u direction and close in v direction or vice versa(cylindrical topology).
3. Surface close in both u and v direction (toroidal topology).

The knot vector is used depending on whether surface is closed or open in that direction as given in Eq(2.14) or Eq(2.15).

3.3 MINIMUM REQUIREMENT

1. The data to be fitted must be ordered in some way, such that contours or other characterstic curves follow on or near the intended shape.

2. If data originates from some measuring system then samples are taken in series or some gridding pattern. Normally any industrial design item does not have random coordinates.

3.4 PRINCIPLE

The surface fitting technique is done by repeated curve fitting. Suppose the data points are arranged as M series of N points, then each series can be approximated by a curve represented by K coefficients. Now there are K series of M points available, each of which can be approximated by a curve represented by L coefficients using exactly the same fitting procedure. The resulting (KXL) coefficients are exactly the control points of the intended approximate surface.

3.5 DESCRIPTION

The following assumptions are made for the input-

1. The data consists of 3D cartesian points(eg x y z coordinates).
2. The data points must be grouped in M series.
3. Each series may consist of different number of data points(N_i ; $i = 1,...,M$).
4. The order in which data points occur in a series are relevant, as is the ordering of series themselves.

On the basis of these data a B-spline surface of order ku,kv will be found that approximates each point $D_{i,j}$ within tolerance T= (T_x T_y T_z) in a sense that for each $D_{i,j}$ there exits a

parametric value pair(u' v') such that

$$\left.\begin{aligned} |S(u',v') - Dx_{i,j}| &\le Tx \\ |S(u',v') - Dy_{i,j}| &\le Ty \\ |S(u',v') - Dz_{i,j}| &\le Tz \end{aligned}\right\} \qquad (3.4)$$

Tx Ty and Tz may be assigned some positive real value. T=(0 0 0) would result in interpolating B-spline surface that passes through all data points.

3.6 CURVE FIT

Initially the x-, y- and z- components of $D_{i,j}$ are fitted independently by single valued B-spline function X(u), Y(u) and Z(u) of orders ku defined on a knot vector as given in Eq(2.14) or Eq(2.15). (Note: N≥ ku or 1≤ N < ku, where N is total number of data is not pre requisite for the fit. These underdetermined cases will be solved appropriately as described later in this chapter).

The B-spline function X(u) with the lowest possible number of control points is fitted to the data. Initially number of control points nu is assumed to be ku. If X(u) fails to approximate the value Dx_i within tolerance Of 1/2 Tx then nu is increased by 1 and fit is retried. So when nu = N, exact data interpolation is obtained with all data points are on the curve. Thus a satisfactory fit is obtained with smallest possible nu existing in the range ku ≤ nu ≤ N.

Fitting of

$$X(u) = \sum_{i=1}^{nu} Px_i \; N_{i,ku}(u) \qquad (3.5)$$

to the values $Dx_i, \ldots, Dx_n$ proceeds as follows -

The knot vector for u parameter is setup for B-spline curve of order ku and of control points nu. To each data point Dx_i, a parameter value tj is assigned in the range [umin, umax]. The fit function is formulated of a function X(u) to data points Dx_j such that

$$\sum_{i=1}^{nu} W_j(X(t_j) - Dx_j)^2 \qquad (3.6)$$

is minimal.

Wj is a weight factor associated with each data point Dj.

But, the parameter value tj has to be found out in some way such that the parameter is distributed in some average pattern within the range [umin, umax].

3.7 CHOICE OF t_j

A minimal requirement for tj is that $t_{j+1} > t_j$ for $1 \leq j < N$ and there is at least one t in each nonzero interval span$[u_i, u_{i+1}]$ for i= ku..,nu. This still leaves many different possible choice for tj.

Here, we consider distribution pattern suggested by Vergeest[12] which gives regular spacing of t values within the parameter range. Suppose that a vector for u parameter has been set up for order ku and nu number of control points then there are nu-ku+1 knot intervals.

First, nu locations s_i are determined from-

$$s_i = \frac{u_{i+1} + u_{i+2} + \ldots + u_{i+k-1}}{ku - 1}, \quad i=1,\ldots,nu \qquad (3.7)$$

The s_i are scaled and shifted to produce s'_1 and s'_{nu} which will be at extremes of u_{ku} and u_{nu+1}.

$$s'_1 = s_i/r - d\ ,\ i= 1,..\ nu. \qquad (3.8)$$

where

$$r = \frac{s_{nu} - s_1}{u_{nu+1} - u_{ku}} \tag{3.9}$$

$$d = \frac{s_1}{r} - u_{ku} \tag{3.10}$$

Total number of data points N is larger than or equal to nu ; therefore if

$$A = \frac{(n_u - 1)}{(N - 1)} \tag{3.11}$$

then, $0 \le A \le 1$.

Finally, t_j is taken as follows:

$$t_j = s'_m + (A(j-1)+1-m)(s'_{m+1} - s'_m) \tag{3.13}$$

where

$$m = \mathrm{INT}(A(j-1)) + 1 \tag{3.14}$$

and INT(a) is the largest integer less than or equal to a (floor function of C -language) .

The above distribution guarantees that t_1 and t_n are located at the interval bounds and there at least lies a nonzero value within each interval,which is supposedly an appropriate condition .

If the weights W_j in Eq(3.6) are taken identical and equal to 1, then with the knot vector and t_j parameter described above it can be verified that there exists an unique set of $Px_1 .. Px_{nu}$ that solves Eq(3.6). The vector form of Eq(3.6) can be written as -

$$\begin{bmatrix} N_{1,ku}(t_1) \ldots\ldots N_{nu,ku}(t_1) \\ N_{1,ku}(t_2) \ldots\ldots N_{nu,ku}(t_2) \\ \ldots \qquad\qquad \ldots \\ N_{1,ku}(t_N) \ldots\ldots N_{nu,ku}(t_N) \end{bmatrix} \begin{bmatrix} Px_1 \\ \vdots \\ Px_{nu} \end{bmatrix} - \begin{bmatrix} Dx_1 \\ Dx_2 \\ \vdots \\ Dx_N \end{bmatrix} \tag{3.14}$$

The expression 3.14 has minimal length(i.e. sum of squares of components has the minimum value). The above matrix is of NX1 size and has many zeroes if ku << nu << N due to local support of b-spline function.

$$N_{i,ku} = 0 \text{ if } t_j \notin [u_i, u_{i+k}] \tag{3.15}$$

The above matrix can be solved as given below,

Differentiating Eq(3.6) for least square error and equating to zero gives the solution as

$$[B^T][B][P] = [B^T][D] \tag{3.16}$$

where

[B] is the basis function matrix given in Eq(3.14).

Eq(3.16) becomes

$$[A][P] = [e] \tag{3.17}$$

Now, [A] is a positive, semi-definite, symmetric matrix. So this makes the system suited for solution by Cholesky factorisation method[18]. The solution gives the control points Pxi.

The same procedure is followed to obtain y(u) and z(u) to approximate Dy and Dz within tolerance 1/2Ty and 1/2Tz respectively. But if one or more components deviate too much from the data ,nu is incremented for all 3 fits with X(u), y(u) and z(u). At this point we have obtained a set of control points P(i) to draw a B-spline curve of order ku.

$$P(u) = \sum_{i=1}^{nu} P_i \, N_{i,ku}(u) \tag{3.18}$$

3.8 RE-REPRESENTATION OF B-SPLINE CURVE

For each of the M rows of data points a b_spline curve $r_j(u)$ can be fitted in the way described above. If different nu_j's are obtained for different rows of data, a choice has to be made for one single value to keep knot vectors identical for all rows. The following choice is made here -

$$numax = \text{Maximum of } nu_1, nu_2, \ldots, nu_M. \qquad (3.11)$$

3.9 UNDER-DETERMINED SYSTEM

1. If $numax > N_j$ the system gets under-determined[12]. The following method to represent the curve removes the above problem.

$numax$ locations e_{ij} on curve $r_j(u)$ are evaluated at parameter value t_i, $i= 1..numax$ and t_i are newly determined for $numax$ control points as per Eq(3.7) to Eq(3.13). Then locations e_{ij} are regarded as $numax$ data points to B-spline curve of order k_u to be fitted or interpolated.

The above method keeps the original fitted curve principally unchanged except forsome extreme cases with large discontinuities.

2. If $N_j < k_u$ the system gets underdetermined -

To handle the above problem , the order is increased to N_j and nu is initialized as N_j. Then the whole procedure is repeated as described above to find $numax$ control points of the curve.

3.10 SURFACE FITTING FROM FITTED CURVES

We have now obtained M B-spline curves $r_j(u)$ of order k_u with $numax$ control points E_{ij} $(i=1...numax,\ j= 1...M)$. Now we regard these control points as $numax$ rows each consisting of M

points. For each row i we determine B-spline curve $r_i(v)$ of order kv that approaches M points E_{ij} within tolerance 1/2 T and with minimum number of points. Then the same procedure as for u parameter is followed to find nvmax control points for B-spline surface S(u,v) of order ku ,kv. As the curve $r_j(u)$ deviates 1/2T from original data points D_{ij} while curves $r_i(v)$ are closer than 1/2T to control points of curve $r_j(v)$, the surface approximate to original data by tolerance of T [13] except for perhaps some extreme cases. In the current implemententation there is no explicit distance check between $D_{i,j}$ and S.

3.11 SURFACE FITTING FOR DIES

In the past, the die surface model was based on hit and trial method. Despite the computer provides excellent facilities for designing surfaces, in most cases computer graphics proves uneconomical for dies because of the effort involved in developing programs in describing the geometry of the shape. Although the parametric surface can easily be made by means of CAD technique, it is obvious that mathematical expression can not exactly match the real sculptured surface. In order to get more adequate parametric surface, more detail data points should lie within each patch and the surface should be fitted to those points.

The surface fitting technique described above may be useful for following two purposes.

1. **REPRODUCTION OF WORN OUT DIE SURFACE**

When a die surface is worn out, the surface may be required to be reproduced with some alteration. The old surface can be measured by Coordinate Measuring Machine by moving its probe on

the surface; for example 100 data are measured in 10 rows. Taking probe error into account, the original surface can be fitted by an approximated surface for further manipulation and subsequent manufacture.

2. PRODUCTION OF A NEW DIE SURFACE

In spite of development of CAD/CAM system the designer still needs models made of wood, clay, plastic or leaflets because their design idea of product shapes is difficult to define in geometric expression and visualize on a 2D picture plane.

After making those models by artists, the surface is to be generated for NC milling or other computer controlled production processes. The surface fitting technique may help for that in a big way.

The fitting technique is particularly useful for defining dies for forging or pressing car body, complicated mould surfaces, cores in die casting etc.

3.11 IMPLEMENTATION

The fitting of surface has been implemented in HP work station. The data can be simple fed manually key board or through a text file. To design a die sculptured surface,the data can be obtained by a CMM -coordinate measuring machine from a clay model or by 2-D or 3-D digitizer and arranged in a grid pattern for the input.

3.12 CONCLUSION

Because of the B-spline basis function support, the above least square fitting technique is much faster than least square solver E04FDF of Nag library which uses general least square method and therefore our method is more suitable for

interactive surface development. However,the control points found by fits are not easily interpretable generally. Smoothness and closeness to the data is controlled by simple tolerance values presently and interactive manipulation of data point is not implemented in our work. Nevertheless it is expected that above method of surface fitting to be of immense help for die surface modelling.

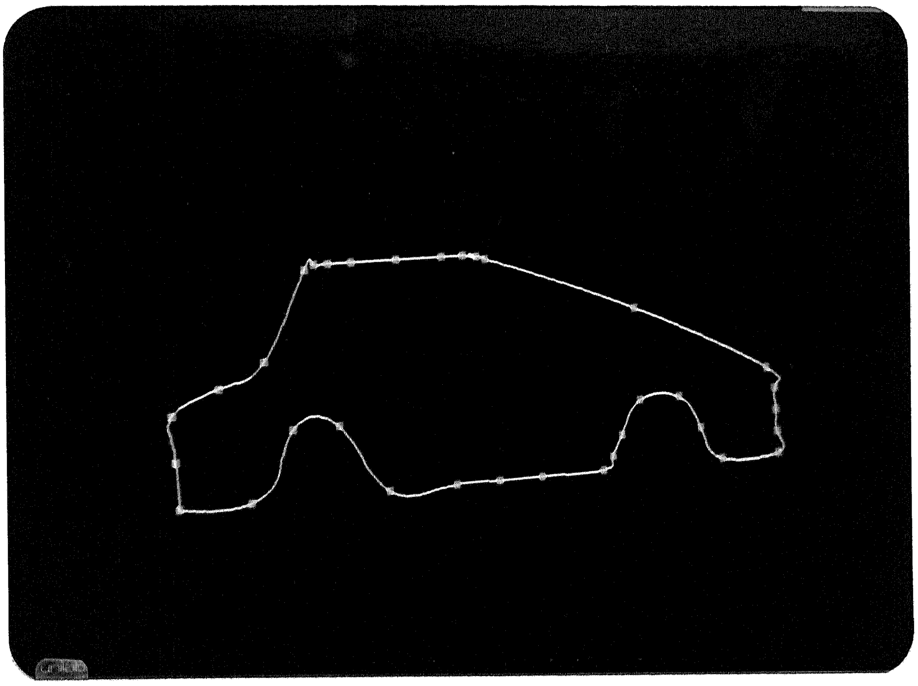


Fig. 3.1 Example of a Curve Fit

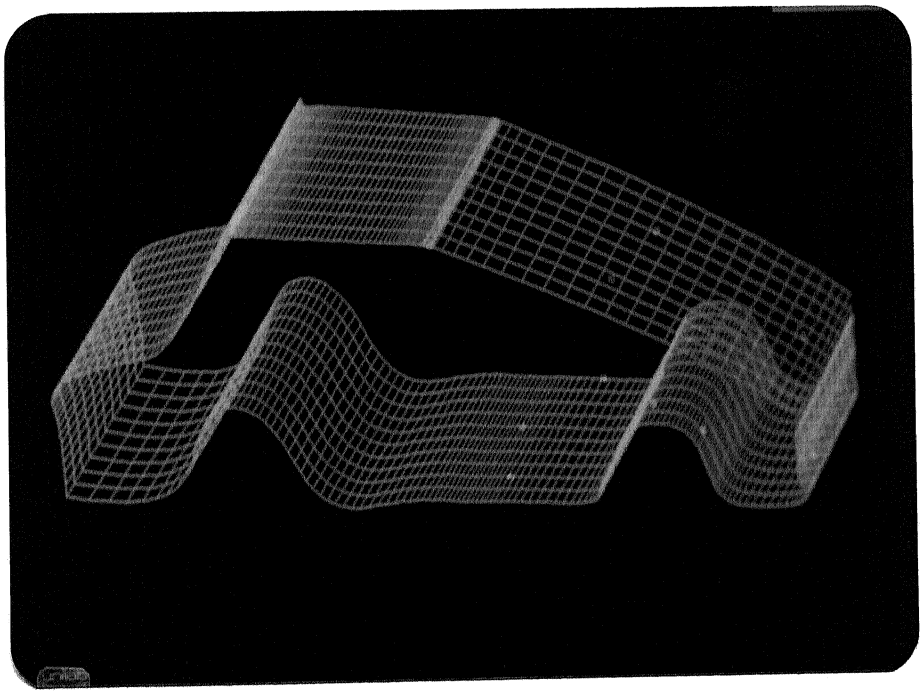


Fig. 3.2 Example of Surface Fit

CHAPTER 4

SWEEP SURFACE MODELLING FOR DIES

4.1 GENERAL INTRODUCTION

A smooth sculptured surface that is best described as trajectory of cross-section curves swept along profile curves is called a sweep surface. It is easy to describe aesthetically appealling sweep surfaces by specifying only 2-D cross sections. The surface S(u,v) can be represented as a series of simple transformation and blending of cross section curves and profile curves. In the proposed model, a sweep surface is regarded as a tuple consisting of cross section curves, profile curves and a sweeping rule. The sweeping rule considered are parallel sweeping ,rotational sweeping, spiral sweeping and synchronized sweeping.

Using the above rules, most of the sculptured surfaces as prescribed by Kishi[2] can be described.

4.2 DESCRIPTION OF SWEEP SURFACE

In engineering drawings, a sweep surface is usually specified by its profile curves and a number of cross section views as shown in Fig. 4.1 . Presented in Fig. 4.2 is the 3D view of the sweep surface. The base plane corresponds to Fig 4.1(b) and the first left cross section plane represents A-A' section in Fig. 4.1(d). The section plane can be assumed a solid plate on which section curve p_0p_1 is drawn and the plate is moved in space so that $b_0(v)$ passes through p_0 while the plate is vertical.

A sweeping rule has to be specified to obtain a unique motion of section plane. If the plate moves in such a way that the flow rates on the two boundary curves are the same, e.g. section plane intersect at the same percentage of the length of the boundary curves, then the surface is called synchronized sweep.

The above sweeping rule guarantees that first boundary curve $b_0(v)$ always passes through p_0 of the moving plate, but the second boundary curve $b_1(v)$ does not in general intersect the plate at p_1 as depicted in Fig 4.3. Thus it is necessary to correct the original section so that the initial p_1 is brought to the current intersection point. The corrected trajectory of the first section curve defines a sweep surface. By repeating the same steps for second cross section curve another sweep surface is obtained. Finally the two sweep surfaces are blended according to a suitable blending rule to obtain the final sweep surface.

4.3 MATHEMATICAL REPRESENTATION

A mathematical model of surface is a mapping from a 2D domain(of parameter u,v) to an E^3 space. Our goal is to obtain a mapping function $r(u,v)$ from the description of two boundary curves and two section curves . The following steps give the mathematical replica of the descriptive procedure of the previous section.

4.4 COORDINATE FRAMES AND CURVE EQUATION

The two bounded curves are represented as parametric curve equations with respect to a 'profile parameter' v:

$$b_1(v) = (x_o(v)\ y_o(v)\ z_o(v)),\quad o \le v \le 1 \tag{4.1}$$

$$b_2(v) = (x_1(v)\ y_1(v)\ z_1(v)),\quad o \le v \le 1 \tag{4.2}$$

The requirement here is that mapping from $v \in [o.1]$ to $b(v)$ is regular and non-singular, that is one to one mapping is required.

Let IV(v) in Fig. 4.3 denote the vector from $b_1(v)$ to $b_o(v)$ on x-y plane. The angle $\theta(v)$ between x axis and 2D vector IV(v) becomes function of v and is given by,

$$\theta(v) = a\tan2(y_o(v) - y_1(v)\ ,\ x_o(v) - x_1(v)) \tag{4.3}$$

In order to have a well defined intersection angle $\theta(v)$ the projected images of two boundary curves are not allowed to intersect with each other.

Each section curve is represented as parametric curve s(u) with $o \le u \le 1$ (section curve parameter u). Also specified in fig 4.5 are coordinates of guide point G through which $b_o(v)$ passes. The intersection vector IV(v) appears on the section plane and the angle ϕ between x-axis and the intersection vector. The guide point is as follows-

$$G = (g_x\ \ g_y\ \ 0) = s(o) \tag{4.4}$$

But it is always possible to define the section coordinate frame such that G = (0 0 0) and $\phi = 180^o$ which makes the transformation matrix much simpler.

4.5 SWEEP TRANSFORMATION

The following sequence of coordinate transformations bring the section plane of x-y frame to a 3 dimensional location

of a section curve[14].

1. Translate so that guide point moves to origin.

2. Rotate - through an angle ϕ around z axis.

3. Rotate - through an angle -90 degree around x-axis.

4. Rotate by $\theta(v)$ angle around z axis.

5. Translate so that G moves back to $b_o(v)$.

The 3 rotations from the core of the sweep transformation can be conveniently expressed in matrix form.

$$Sweep(\phi, \theta(v)) = Rot(z, -\phi)\ Rot(x, -90)\ Rot(z, \theta(v))$$

which can be expressed in mathematical form [8]

$$\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta(v) & \sin\theta(v) & 0 \\ -\sin\theta(v) & \cos\theta(v) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.5)$$

$$= \begin{bmatrix} \cos\phi\ \cos\theta(v) & \cos\phi\ \sin\theta(v) & \sin\phi \\ \sin\phi\ \cos\theta(v) & \sin\phi\ \sin\theta(v) & -\cos\phi \\ -\sin\theta(v) & \cos\theta(v) & 0 \end{bmatrix} \quad (4.6)$$

Where ϕ and $\theta(v)$ are intersection angles on the section plane and base plane, respectively.

If ϕ is chosen as 180^o, the sweep transformation matrix simplifies to-

$$Sweep(\theta(v)) = \begin{bmatrix} -\cos\theta(v) & -\sin\theta(v) & 0 \\ 0 & 0 & 1 \\ -\sin\theta(v) & \cos\theta(v) & 0 \end{bmatrix} \quad (4.7)$$

In other words the sweep surface $P(u,v)$ is obtained by sweeping a section curve $s(u)$ along a boundary curve $b_o(v)$ and is expressed as the following sweep transformation.

$$P(u,v) = (s(u) - G)Sweep(\phi, \theta(v)) + b_o(v); \quad 0 \le u, v \le 1 \quad (4.8)$$

If guide point G becomes the origin and intersection

vector IV(v) is in minus x-axis direction (*ie*. G=(0 0 0) and ϕ = 180^o) ,then sweep surface equation becomes

$$P(u,v) = s(u)Sweep(\theta(v)) + b_o(v) \;;\; 0 \le u,v \le 1. \qquad (4.9)$$

4.6 CORRECTION FOR SURFACE BOUNDARIES

As described earlier, the end of section curve in general is away from boundary curve $b_1(v)$, the curve has to be brought to $b_1(v)$ by some way for the completion of the surface. The following method is suggested by Choi and Lee[14].

First, the section curve is scaled so that distance between the 'start point' p(o.v) and 'end point' p(1,v) becomes equal to the distance between two boundary curves. Second, a section curve is obtained so that p(o,v) coincides with $b_o(v)$. Third, another section curve is obtained by moving P(1,v) to $b_1(v)$; and finally blending two two resulting section curves by using a blending function. The scaling factor f(v) is defined as

$$f(v) = \frac{|\, b_o(v) - b_1(v) \,|}{|\, s(o) - s(1) \,|} \qquad (4.10)$$

The corrected intermediate section curve q(u,v) is then expressed as a blend of two scaled and translated intermediate section curves :

$$q(u,v) = \alpha(u)\{ f(v)[p(u,v) - p(0,v)] + b_o(v)\}$$
$$+ B(u)\{f(v)[p(u,v) - p(1,v)]+b_1(v)\}. \qquad (4.11)$$

where α and B are two blending functions.

4.7 BLENDING

Similarly, two section curves can be blended by a suitable blending function. The equation of the blended sweep

surface is expressed as :

$$r(u,v) = \alpha(v)q_0(u,v) + B(v)\, q_1(u,v)$$
$$= \alpha(v)f_0(v)[s_0(u) - \alpha(u)s_0(o) - B(u)s_0(1)]\text{Sweep}(\phi_0, \theta(v))$$
$$+ B(v)f_1(v)[s_1(u) - \alpha(u)s_1(o) - B(u)s_1(1)]\ \text{Sweep}(\phi_1, \theta(v))$$
$$+[\alpha(u)b_0(v) + B(u)b_1(v)] \ ; \quad 0 \le u,v \le 1. \qquad (4.12)$$

where $s_i(u)$ are equations of section curves $i = 0,1$; $f(v)$ is the correction factor as given in eq.4.10 . ϕ ,Sweep(ϕ, $\theta(v)$) are as defined earlier. $\alpha(*)$ and $B(*)$ are two blending function chosen such that $\alpha(o) = B(1) = 1$, $\alpha(1) = B(0) = 0$, and $\alpha(u)+B(u) = \alpha(v)+B(u) = 1$.

The choice of the blending function $\alpha(*)$ and $B(*)$ in eq(4.12) affects the quality of the surface. The simplest choice is linear blending function:

$$\alpha(u) = 1- u \ ; \ B(u) = u; \qquad (4.13)$$

Another blending function can be chosen called Hermite blending function[16]; given as :

$$\alpha(u) = 1 - 3u^2 + 2u^2 \ ; \qquad B(u) = 1 - \alpha(u); \qquad (4.14)$$

The above blending function is suitable for providing tangent continuity and therefore can be used when a sweep surface is constructed from network of boundary and section curves.

The above model describes a synchronized sweeping surface.

4.8 GENERAL SWEEP SURFACE

For the general sweep surfaces, the concept of guide curve is introduced. A guide curve could either be spine curve or boundary curve. The boundary curve should not necessarily lie on the surface. For example, if the section curve is a closed

curve ,then the centre of cross section of the closed curve can be taken as guide curve. A parametric form of guide curve is:

$$g(v) = (X_g(v)\ Y_g(v)\ Z_g(v)) \tag{4.15}$$

Derivative of the guide curve is :

$$dg(v)/dv = (\dot{X}_g(V)\ \dot{Y}_g(V)\ \dot{Z}_g(V)) \tag{4.16}$$

The intersection angle $\theta(v)$ under different sweeping rules are given as follows:

1. $\theta(v) = \theta_o$ (Parallel sweep) (4.17)

2. $\theta(v) = \mathrm{atan2}(Y_g(v) - a_y,\ X_g(v) - a_x)$ (4.18)
(Rotational sweep)

$a = (a_x\ a_y\ o)$; coordinate of the axis of roatation.

3. $\theta(v) = \mathrm{a\ tan2}(\dot{X}_g(v)\ ,-\dot{Y}_g(v))$ (4.19)
(Spined sweep)

4. $\theta(v) = \mathrm{a\ tan2}(Y_o(v) - Y_1(v),\ X_o(v) - X_1(v))$ (4.20)
(synchronized sweep)

For a given value of v the profile parameter and guide curve g(v) the sweep transformation of Eq.(4.9) can be modified as:

$$P(u,v) = (s(u) - G)\mathrm{Sweep}(\phi\ ,\theta(v)) + g(v) \tag{4.21}$$

4.9 GENERAL CORRECTION TRANSFORMATION

The points on the intermediate section plane at which the two boundary curves intersect with it are determined by numerically solving the following equations[18].

$$\overline{b_o}(s).\overline{n(v)} = \overline{g}(v).\overline{n}(v) \tag{4.22}$$

$$\overline{b_o}(t).\overline{n(v)} = \overline{g}(v).\overline{n}(v) \tag{4.23}$$

where n(v) is the normal vector of section plane.

EXAMPLE OF SOLUTION:

The parametric form of B-spline or Bezier curve are given by Cohen and Riesenfeld[16]. For example Bezier curve can be

deduced in the form -

$$p(u) = [U][N][G] \quad (4.24)$$

where ,

$$[U] = [u^n \ u^{n-1} \ldots\ldots u \ 1] \quad (4.25)$$

$$N_{i+1,\, j+1}\Big]_{i,j=0}^{n} = \begin{cases} ({}^nC_j)\ ({}^{n-j}C_{n-i-j})\ (-1)^{n-i-j} & 0 \le i+j \le n \\ 0 & \text{otherwise.} \end{cases} \quad (4.26)$$

$$[G]^T = [\ p_0\ p_1\ \ldots\ldots p_n] \quad (4.27)$$

Using equations Eq(4.21) to Eq(4.27), a polynomial function can be formed whose roots may be real and distinct, real or equal or complex roots. So, the solution can be found out by Graff's root squaring method[18].

Let s^* and t^* be the solutions of the equations. Then scaling factor $f(v)$ for general sweeping rule is given by

$$f(v) = |b_0(s^*) - b_1(t^*)| / |\ s(0) - s(1)\ | \quad (4.28)$$

The final generalized sweep surface can be mathematically expressed in the form:

$$\begin{aligned} r(u,v) = {} & \alpha(v) f_0(v)[s_0(u) - \alpha(u)s_0(u) - B(u)s_0(1)]\mathrm{Sweep}(\phi_0, e(v)) \\ & + B(v) f_1(v)[s_1(u) - \alpha(u)s_1(0) - B(u)s_1(1)]\mathrm{Sweep}(\phi_1, e(v)) \\ & + [\alpha(u) b_0(s^*) + B(u)\ b_1(t^*)] \ ; \quad 0 \le u,v \le 1. \end{aligned} \quad (4.29)$$

In summary, a sweep surface has one guide curve, up two boundary curves and one or two sweep curves.

For completeness, it may be noted that the die sculptured surface of fig 2.2 can be generated using sweep technique and different combination of boundary and section curve or curves.

The above surface model technique could be used for forging, moulding and casting dies.

without twisting or bending.

The basic concept of extrusion die geometry is shown in fig(4.).The sector of the billet has been mapped onto corresponding sector on each side with same extrusion ratio being maintained. Hence,

Area OPQ / Area OP'Q' = R = Area OPR / Area OP'R'

where R is the extrusion ratio.Gunasekara[3] suggested that spline of any order can be fitted to the entry and exit sections to obtain a streamline dies.

But, the area mapping technique fails for a re-entrant type die geometry as shown in fig.(4.8). Area mapping technique is transformed to perimeter technique by use of Stokes theorem[10] which means the perimeter of the billet has to be mapped to the perimeter of the extruded product. This solution is more generalized and applicable even to re-entrant section.

In our work the following method of cylindrical sweep surface has been used to describe extrusion die geometries:

A. SYMMETRIC CASE:

4.112 Product Section :

The coordinates of extruded product is decided first in two dimensional (x-y) plane. If the product section has all straight line edges then a B-spline of order 2 is fitted to the control points to form a close curve. If the section has curves a higher (3rd or 4th) order spline is fitted.

4.113 Symmetric Axis:

On the product boundary axis of symmetric has to be decided. The axis of symmetry is taken to be the x and y axes through the centroid of the product. Centroid is decided as:

$$X_c = \frac{l_1 X_1 + l_2 X_2 + \ldots + l_n X_n}{l_1 + l_2 + \ldots + l_n} \tag{4.30}$$

$$Y_c = \frac{l_1 Y_1 + l_2 Y_2 + \ldots + l_n X_n}{l_1 + l_2 + \ldots + l_n} \tag{4.31}$$

where l_1, l_2 .. etc are the segment length on the product section, and X_1, Y_1 ..etc are the coordinates of the centroid of each segment measured from the centre.

4.114 Die Parameter

The die parameter such as billet diameter and die length are decided from analysis for optimum design and can be fed as input. The billet diameter is drawn taking centroid of the product as the centre.

4.115 Surface Construction

1. An adequate number of characterstic points on the exit curve are selected.

2. The mapping points on the billet are determined by connecting a straight line between the centre point (X_c, Y_c) and the characteristic points on the exit boundary.

3. V-curve construction (Axial direction)

It is assumed that the axial profile of the die are streamlines which can be designed with flexibility. To draw a streamline, two more control points are decided between product section's characterstic points 'P_i' and corresponding mapped point 'Q_i' on 'billet diameter', the derivative of the B-spline at P_i and Q_i are

$$P_i' = 3(P_i - E_i)\ ,\ Q_i' = 3(B_i - Q_i) \tag{4.32}$$

where B_i and E_i are the Z coordinates of the control points corresponding to Q_i and P_i (X,Y) coordinate fig(4.9). Assuming

axial entry and exit ,(tangent is zero) z coordinates are taken as 1/4(die length) from P_i and Q_i. The streamline constructed by Q_i, B_i, E_i, P_i are shown in fig(4.9).

Now, four control points are obtained for each stream line. The stream line can be swept on two cylindrical curves for 360^o to produce a extrusion die geometry. This can be done in simpler manner. The four points are decided through all characterstic points and a close B-spline surface can be fitted through these points.

4.116 Unsymmetrical Case:

For unsymmetric section with re-entrant shape the perimeter may not map exactly as described before. We suggest for those cases that node 1 of the product cross section should start from a point on the billet which intersects the positive y-axis and the rest of the procedure follows as above.

However, a detailed analysis has to be done on how the material flow occurs in these types of surfaces to decide the smooth stream lines.

Some of the examples are given at the end for illustration.

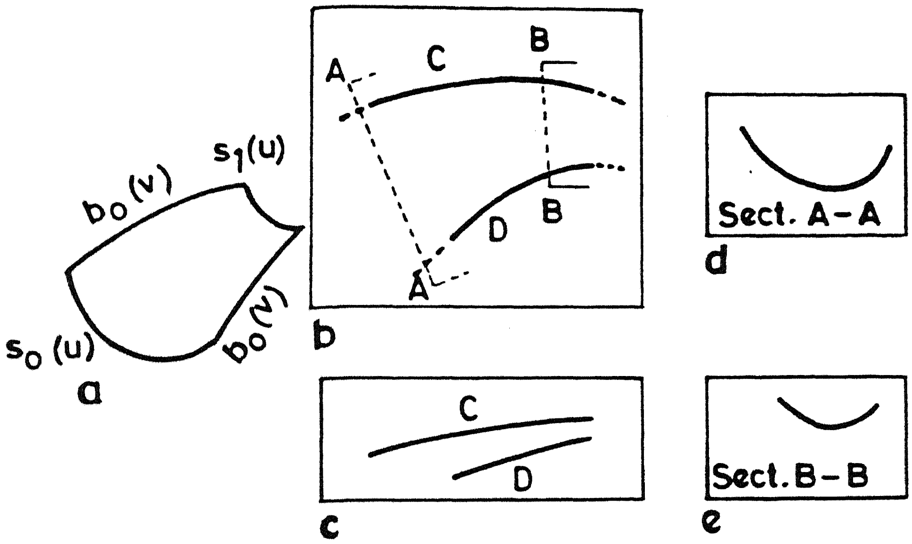


Fig. 4.1 Description of sweep surface[14]

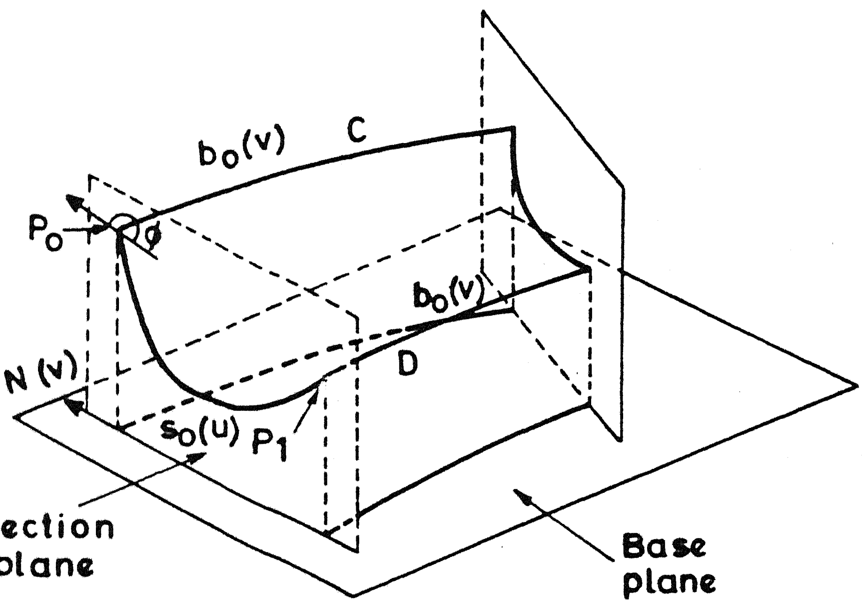


Fig. 4.2 3D view of sweep surface[14]

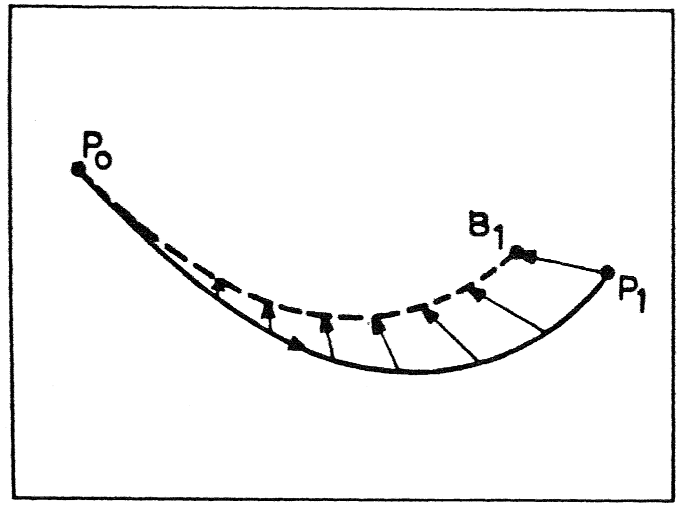


Fig. 4.3 Correction of intermediate section curve.[14]

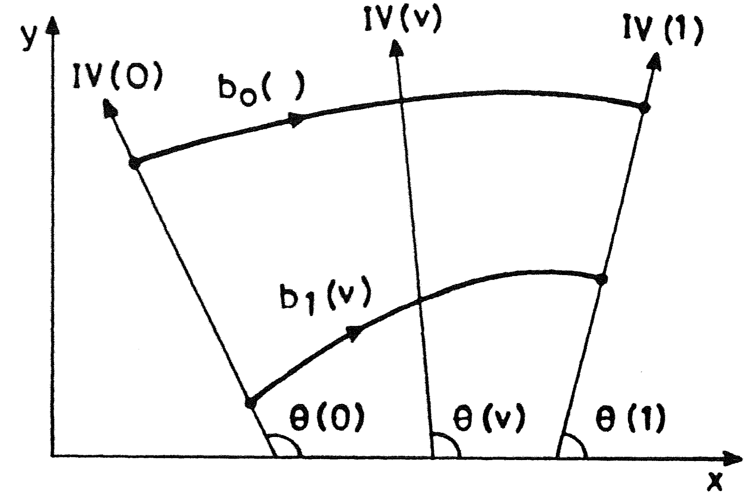


Fig. 4.4 Intersection vectors and intersection angles on base plane.[14]

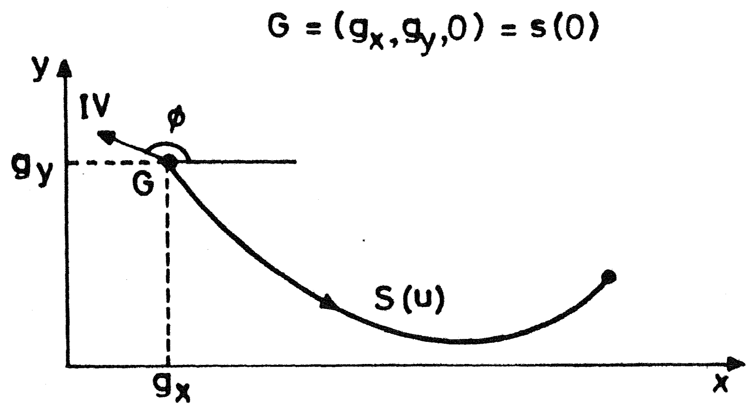
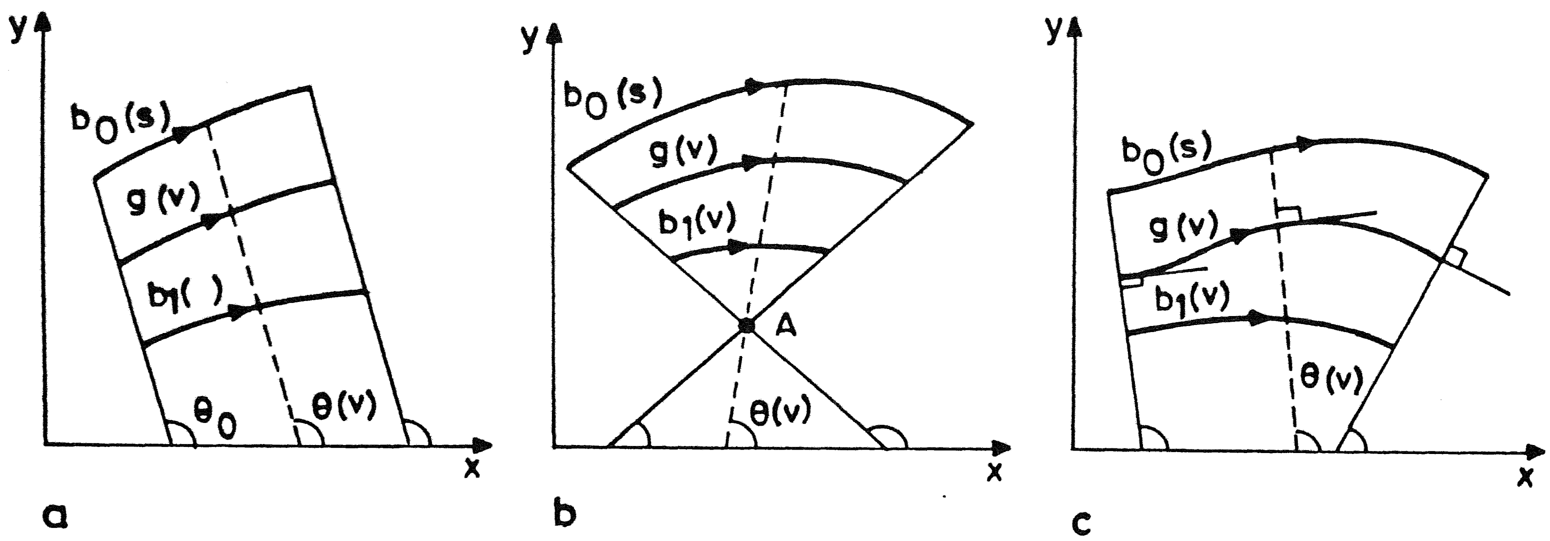


Fig. 4.5 Section coordinate system for synchronised sweeping.[14]



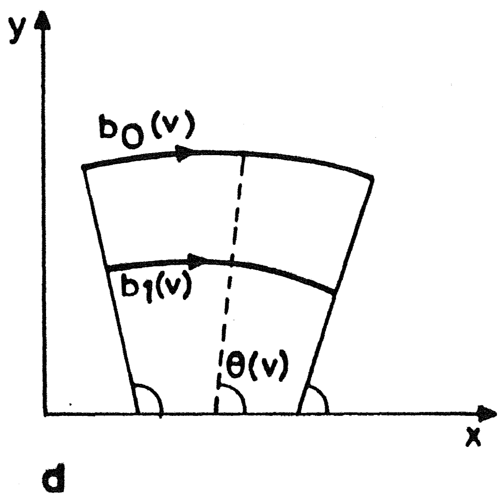


Fig.4.6 Sweeping rules: (a) parallel (b) rotational (c) spined (d) synchronized.[14]

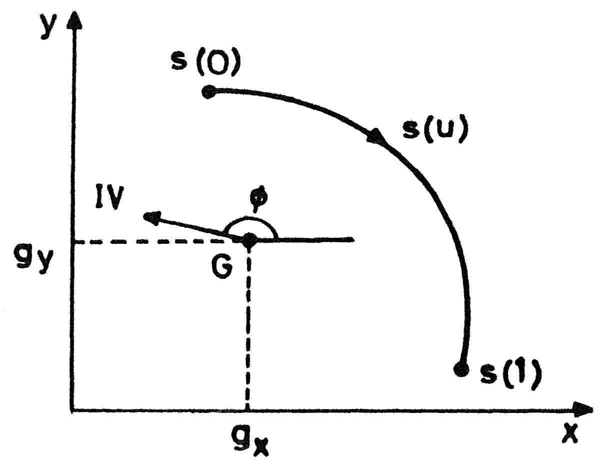


Fig.4.7 Section coordinate system for a general sweeping.[14]

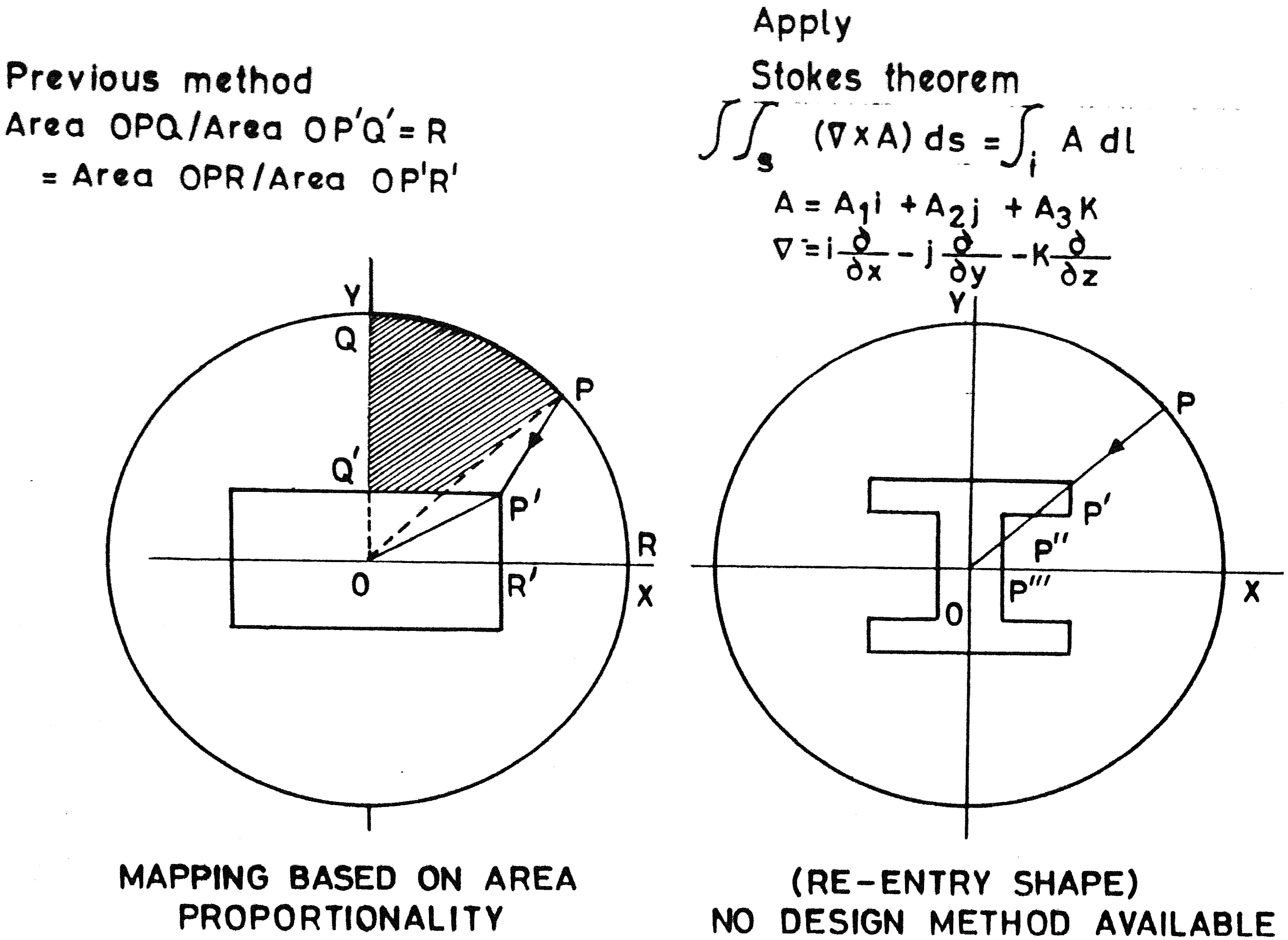


Fig. 4.8 Limitations of previous methods of die design.

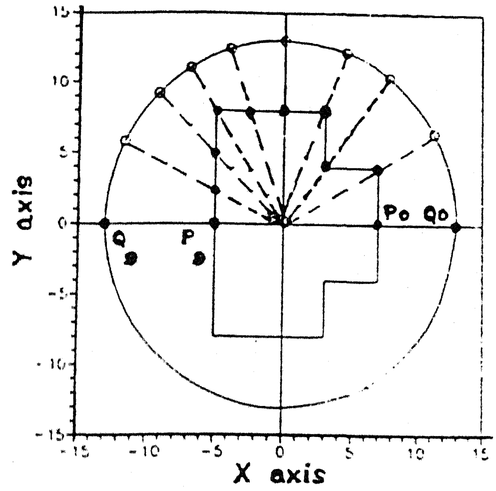


fig. 4.8 Characteristic Points and Mapping Points

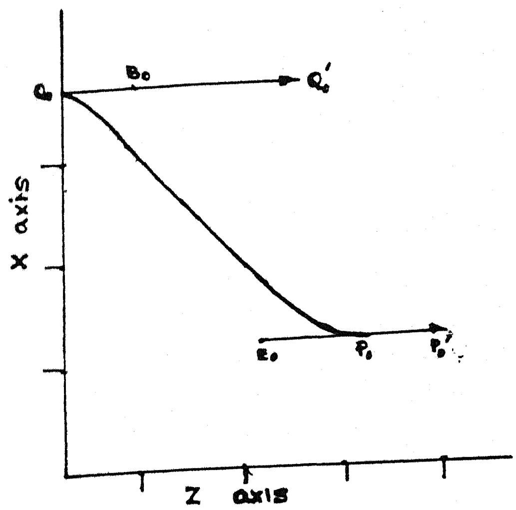


fig. 4.9 The Streamline Profiles

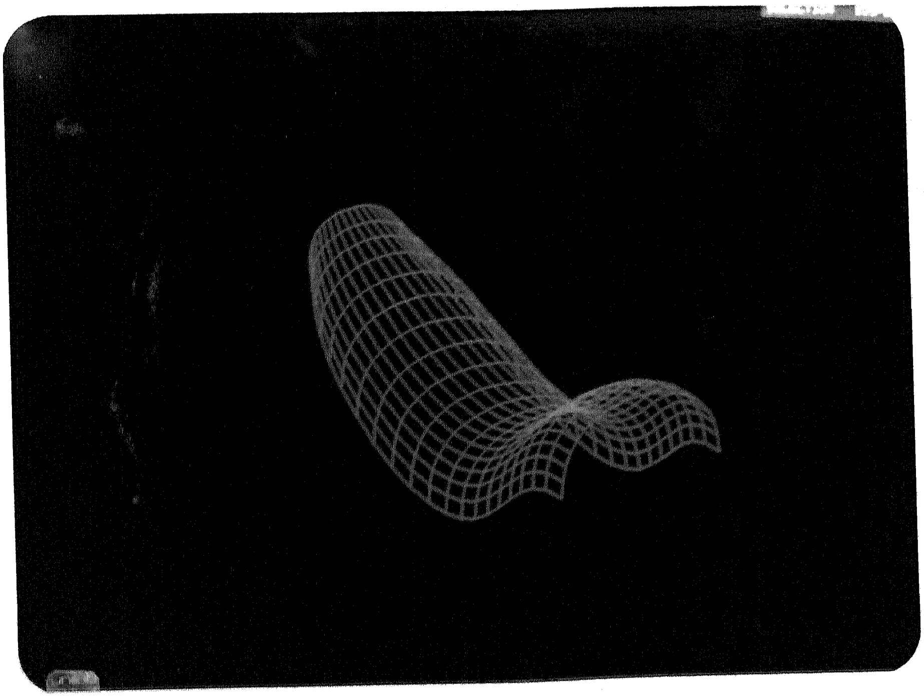


fig. 4.10 Example of Synchronized Sweep Surface

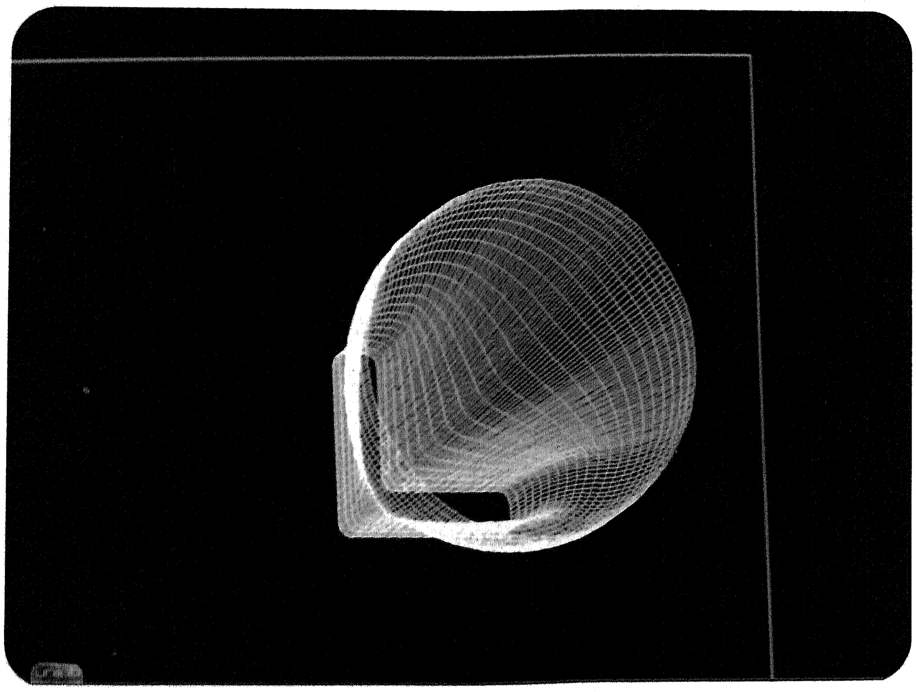
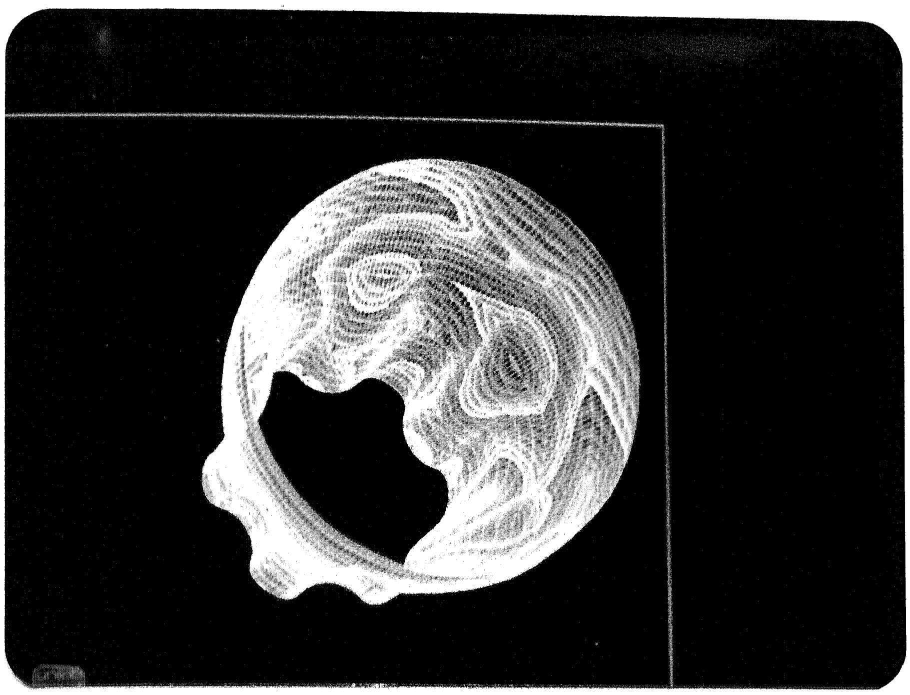


fig. 4.11 Example of Cylindrical Sweep Surface



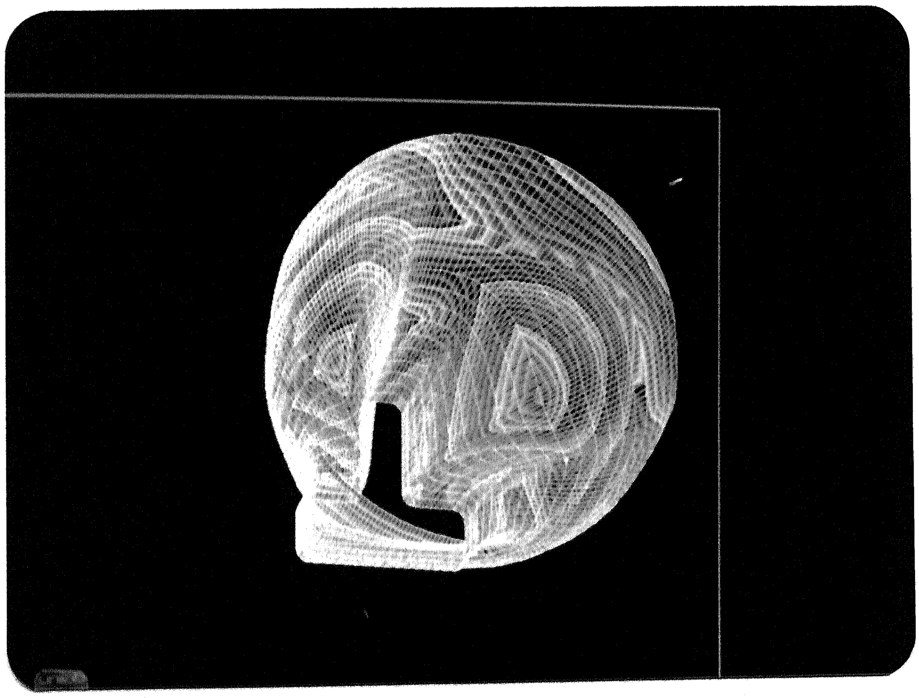


fig. 4.12 Example of rendering for extrusion die surface

CHAPTER 5

CONNECTING CURVES AND SURFACES

5.1 INTRODUCTION

Complicated die surfaces can be designed by conventional method of supplying control points or data points. But the efficiency of the designer will be less. Therefore some supporting methods like gluing, blending or intersection should be implemented in the software to facilitate the designer.

Here we attempt a method of connecting B-spline curves end to end and connecting B-spline surface patches at the boundary. Our method ensures tangency or curvature continuity of the connecting curve or surface patches as required by the designer.

5.2 CONNECTING B-SPLINE CURVES

A complicated curve may consist of linear as well as curved part. As B-spline curve has everywhere C^{k-2} continuity, (k is the order of the curve)an choice of order decides the positional, tangency or curvature continuity. For example order 2 joins the control polygon by linear segments, order 3 generates a curve inside the control polygon with tangent and order 4 generates curve with curvature continuity. Most of the die surface at best need curvature continuity either on the surface or on the boundary curve.

The following steps are suggested to draw curve separately and to connect them end to end.

1. The maximum order required for any individual curve is chosen

as order of the curve.

2. The whole curve is drawn roughly on paper.

3. The curve is separated into either linear, curve segmentor a combination of both segments.

4. The control points are input interactively keeping in mind that minimum number of control points for each curve should be equal or greater than the order of the curve.

5. If the curves contains both straight line or curved segment then there must be two control points at each end of the straight line and in addition, one or two control points are added outside depending on tangency or curvature continuity required.

6 The subsequent segments initial control point should coincide with the last point of the curve.

5.3 CONCATENATING CONTROL POINTS AND KNOT VECTOR

Concatenating control is done by combining all the points and deleting the redundant points in the array.

The individual knot vectors for different segment has to be concatenated to make it a single curve. The following simple algorithm is implemented in our software to combine knot vectors into one.

1. The entire last group of equal values are deleted from the first knot vector.

2. The first element from first group of equal values in the second knot vector is deleted.

3.' n'is added is added to each element in the second knot vector, where 'n' is the value of the elements removed.

4. The knot vector is considered as one.

Example:

knot 1 = [0, 0, 0, 1, 2, 3, 4, 4, 4]

knot 2 = [0, 0, 0, 1, 1, 1]

step1: knot vector # 1 becomes [0, 0, 0, 1, 2, 3]

step2: knot vector #2 becomes [0, 0, 1, 1, 1]

step3: knot vector # 2 becomes [4, 4, 5, 5, 5]

step4: The new knot vector is [0, 0, 0, 1, 2, 3, 4, 4, 5, 5, 5] .

The above procedure has been implemented in the software so that the designer only has to give control points and the order for separate segments interactively.

5.4 SURFACE CONNECTION

5.4.1 Necessary Condition

Our method connects a boundary of a given B-spline surface with the boundary of another surface. The two surface and the connecting patch can be unified while keeping tangency or curvature continuity everywhere along the joints.

Suppose two surfaces **a** and **b** are given each having at least one edge (none of them is doubly closedlike a torus). We aim to to construct a patch c which satisfies following conditions.

1. c = c(u,v) is a biparametric B-spline surface.

2. c has either plane topology or cylindrical topology.

3. One of the c's edges coincides within some tolerance with one of **a**'s edges. The contact curve c_a will fix the u-part of the parameterization of c: curve $c_a = c(u,0)$.

4. Across the boundary between a and c there will be curvature (C^2) continuity, although C^1 and C^0continuity can be forced whenever necessary.

5. The speed of variation of the tangent direction or the curvature changes continuously along the curve and is kept inversely proportional to the local distance between surfaces a and b.

6. The average of this speed variation can be controlled by the designer.

7. The condition from 3 to 7 should be also valid for curve cb between surface b and c.

5.4.2 Mathematics for Connecting Patches

The method of generating surface patch for connecting two other surface patches proceeds as given below[13]

1. A net of control points, the B-spline order in u and v direction, two knot vectors(for u and v parameter) are to be chosen to describe two surfaces.

2. c should be closed in the u direction if the relevant edges of a and b are closed curves.

3. The edge c(u,0) will be completely determined by the corresponding row of points for c. A number (D) of points di (i = D) on the boundary of a should be evaluated and be used as interpolation data points.

4. The algorithm will supply 2nd order geometric continuity that ensures lower order C^1 and C^0 continuity.

5.4.3 Determination of Control Points

First of all, we determine a unit tangent vector $\mathbf{t}_i$ and radius of curvature ρ of surface **a** at $\mathbf{d}_i$, from which the geometry of c can be extrapolated, Fig 5.2(a). It is sufficient to determine the osculating circle of some curve perpendicular to the boundary at d_i.Point $\mathbf{d}_i$ and two non-degenerate points(if the points are at least .001mm apart) e_i and f_i on surface **a** near d_i, fig.5.1 are determined approximately with center m_i and radius ρ_i through d_i as follows:

$$g_i = e_i - d_i \tag{5.1}$$

$$h_i = f_i - d_i \tag{5.2}$$

$$k_i = g_i \cdot h_i \tag{5.3}$$

$$\rho_i = \frac{|h_i|^2(|g_i|^2 - k_i)g_i + |g_i|^2(|h_i|^2 - k_i)h_i}{2|g_i \times h_i|} \tag{5.4}$$

$$|\rho_i| = \frac{|g_i|\ |h_i|\ |g_i - h_i|}{2|g_i \times h_i|} \tag{5.5}$$

$$m_i = d_i + \rho_i N_i \tag{5.6}$$

The second term of Eq 5.6 is a vector of length equal to radius of curvature ρ_i in the direction of the normal vector $\mathbf{N}_i$ at d_i[10].

Now a curve c_i is tried to fit at point $\mathbf{d}_i$ with start direction $-g_i$ and with start radius of curvature ρ_i. One of the simplest choice is made for c_i is a cubic open B-spline curve $c_i(v)$ with four fold knot vector(*ie.*order 4) at the beginning (v = 0) of its parameter range. The start properties (at v=0) of $c_i(v)$ are found in terms of its first 3 control points D_i, E_i, F_i :

$$c_i(0) = D_i = d_i \tag{5.7}$$

Differentiating the B-spline basis function with respect v and

putting $v = 0$ the following values are obtained for a 4th order non-rational B-spline[16].

$$\dot{c}_i = d/du\ c_i(0) = 3(E_i - D_i) \tag{5.8}$$

$$\ddot{C}_i = d^2/d^2u\ C_i(0) = 6D_i - 9E_i + 3F_i$$

$$= -6(E_i - D_i) + 3(F_i - E_i) \tag{5.9}$$

The radius of curvature at the start of the curve is[10]:

$$R_i = \frac{|\ \dot{C}_i(0)\ |^3}{|\dot{C}_i(0) \times \ddot{C}_i(0)|} = \frac{3|E_i - D_i|^2}{|F_i - E_i|\ \sin\theta_i} \tag{5.10}$$

or $$\sin\theta_i = \frac{3|E_i - D_i|^2}{\rho_i|F_i - E_i|} \tag{5.11}$$

where θ_i is the angle between (F_i-E_i) and $(E_i - D_i)$. Further constraints are that D_i, E_i and F_i are coplaner with e_i and f_i.

we have imposed the following condition to determine the point E_i.

$$E_i - D_i = \frac{1/4\ R_i\ (d_i - e_i)}{|d_i - e_i|} \tag{5.12}$$

where R_i is any positive number (the choice of R_i will be made later in this section). For given R_i, d_i, e_i and f_i (thus given θ_i) there still remain several choices for θ_i and $|F_i-E_i|$ in 3 dimensional field. We have chosen

$$|F_i - E_i| = |E_i - D_i| = 1/4\ R_i \tag{5.13}$$

$$\sin\theta_i = \frac{3}{4} R_i/\rho_i \tag{5.14}$$

From the above equation point F is determined as follows.

Referring Fig 5.2(b)

$$p_i = h_i - \frac{(g_i . h_i)\ g_i}{(g_i . g_i)} \tag{5.15}$$

$$E_h = E_i + 1/4\ R_i \cos\theta_i\ g_i \tag{5.16}$$

$$F_i = E_h + \frac{p_i}{|p_i|}\ 1/4\ R_i \sin\theta_i \tag{5.17}$$

5.4.4 Choice of constant Ri

The detailed development of the curvature of C_i(starting with e_i) depends largely on R_i. This parameter indicates over which range the curvature is influenced by the geometry of surface. A simple choice can be the local distance between the two surface **a** and **b**, where curve C_i intersects them.

$$R_i = |C_i(max) - C_i(min)| \qquad (5.18)$$

However, the choice R_i can be varied to have control over the connecting surface patch.

The same procedure described above for surface **a** must be performed on the boundary of **b** to determine curve C_b at v = vamx. The same number of data points are evaluated at the edge of edge **b**. Now we have 6 control points for each d_i point on the edges. So a surface can be interpolated taking DX 6 control points and of desired degree.

5.4.5 Special cases

special cases should be treated carefully,e.g. if d_i, e_i and f_i are colinear then radius of curvature becomes infinite. Then the curve C_i will start as a linear interpolation.

5.5 IMPLEMENTATION

The above mathematics is implemented in a program which joins the $v = v_{min}$ of second surface to $v = v_{max}$ of first surface. Three points are chosen near the boundary to be connected very closely to decide the radius of curvature of the osculating circle. However, connecting B-spline surfaces can hardly be performed exactly[13]. But the above method

approximately meets the condition from 3 to 7 and by deciding the number of data points on the boundary of the connecting surface and surface patches are found to be reasonably good.

5.6 USEFULNESS IN DIE SURFACE DESIGN

As mentioned earlier, the above method can be used as toolto increase the efficiency of the surface model by using it for connecting arbitrary surfaces obtained by either control point manipulation or data fitting and making fillets between close or open surfaces or blending two surfaces patches.

5.7 CONCLUSION

The quality of connecting patch is checked by only graphics(i.e. by line drawings). In the program the boundary fits are not cofined by preset tolerance, nor is the interpolation error evaluated after the fit. Hence, the usefulness for the die design can be realised by a NC product model of the shapes.

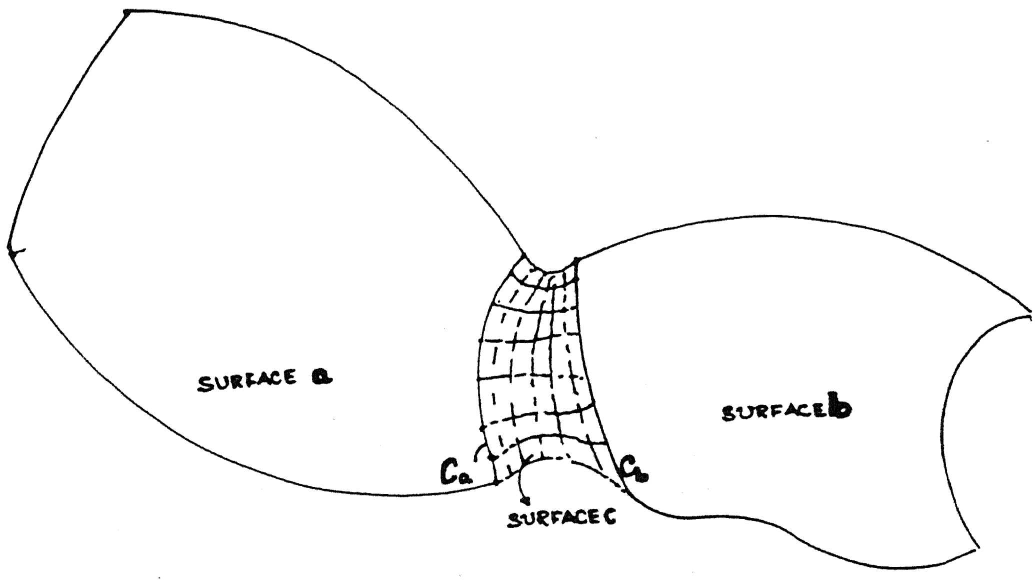


fig. 5.1 Surface c Connects a and b

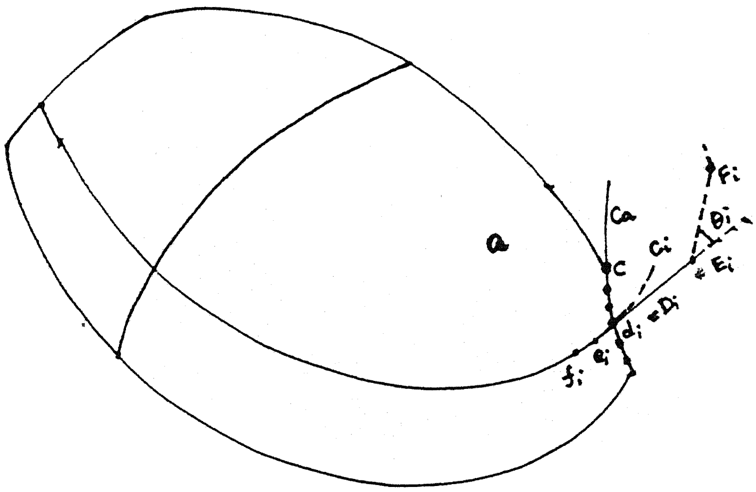


fig. 5.2 Determination of Control Points

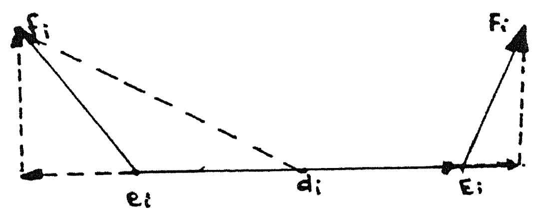


Fig. 5.2(b)

CHAPTER 6

CONCLUSIONS AND SCOPE OF FUTURE WORK

6.1 CONCLUSIONS

In the present work, the main objective was to model die sculptured surfaces using various methods available in literature for surface modelling, data fitting, sweeping of curves to create more complex surface. In this thesis, an attempt has been made to study and implement the above methods to describe various die surface geometry.

First of all it is justified that the choice of parametric equation and specifically that of B-spline is a good choice for die surface modelling.

The second chapter describes a mathematical model for data fitting of B-spline surface. In the implementation, less than 100 data points with maximum 5th degree splines are satisfactorily tried. Theoretically, any number of data points to any order of curve could be fitted. However, the dimension has been limited to 500 data points and 5th degree curve because a curve within 5th degree is sufficient to describe any die sculptured surface and secondly Starbase graphics routines allows maximum of 5th order splines. The surface fitting technique seems to be a very useful method for die surface fitting. The interactive design is left for future scope of work.

Sweep surface technique can really meet the demand for variety of open as well as close die surfaces particularly for open forging, die casting, injection moulding and cores for

casting.

Connecting arbitrary B-spline is attempted only to increase efficiency of surface model, however, the same is not found very useful for the surface modelling of dies.

The software is presented with menu based program. Basically the input is either provided by text file or through keyboard. The surface model can be done in wire frame model or in solid model. The view can be seen from various angle by changing camera position. Some surface model are tried using shading and light source.

The software is incomplete in the sense that it is not yet user friendly and interactive design is not implemented for all modules.

6.2 SCOPE OF FUTURE WORK

6.2.1 Trimming

Trimming is an important tool for generating complex 3D surface for a die. A trimmed surface is essentially a regular tensor product surface but certain areas are marked as invalid or invisible. These areas are usually defined by dense polygons within the u,v domain of the patch. Starbase graphics has trimming of 3D B-spline surface routine which can be simply defined by control points as described for a surface. However, it does not allow multiple trimming surface. Multiple trimming could be a future scope of work.

6.2.2 Offset of 3D Sculptured Surface

6.2.2 Offset of 3D Sculptured Surface

The offset of the whole 3D surface module is vital for die sculptured surfaces for adding tolerances etc. The offset technique for 3D surface is still a state of art subject[19]. The offset of the surfaces can be carried out by shifting the control polygon in normal direction. But simple shifting control points does not necessarily offset 3D surface uniformly. Further analysis could be carried out to find out a well defined normal of the surface for offset purpose.

6.2.3 Intersection And Blending

The intersection of surface patches and consequent blending will be very useful for designing various complicated surfaces. Particularly, it will be helpful for the design of injection moulding die surface which consist of many intricate geometries. This work is being presently carried out by various researchers[20].

6.2.4 Machining of parametric die surface

To manufacture a die surface defined by parametric equation is not very difficult with the advent of powerful CNC machines. To machine a 3D surface a minimum $3\frac{1}{2}$ axis CNC machine is required for milling operation. There are already many software packages readily available to generate NC code for 3D surfaces for example CAM-1, APT, CASPA etc [3]. Some possible problems involve in real machining conditions are as following

1. Overcoming the problems of the sharp corners on the surfaces

2. Scallop Tolerance - To decide the subsequent course of passes of the tool, which should be acceptable between consecutive passes. The scallop tolerance decides total work needed for surface machining for a finished product.

3. Gouging- This is a problem arises due to lack of proper panning of tool path resulting in under cutting in some part.

4. Adaptive path planning- Which considers local optimum paths which are assembled to a continuous global path.

5. Tool and work piece interference: In complex 3D surface machining tool might interfere with the work piece unless path checked clearly.

Taking into account of the above problems the die surface machining by CNC machines for either to produce the die itself or to use EDM could be further point of research for the development of integrated CAD/CAM of dies.

REFERENCES

1. Keys A. Karl, *Innovation in Die Design*, Society of Manufacturing Engineers Publication, 1982, pp. 3-18.
2. Kishi H., "CAD/CAM for Die Making Industry", Manuf. Eng. Vol. 87,No 5, 1981, pp. 90-92.
3. Gunasekara J.S and Hoshino S., "Extrusion of Noncircular Sections through Shaped Dies", Annals of CIRP , Vol. 29, 1980, pp. 141-145.
4. Duncan J.L., "New Direction in Sheet Metal Forming Research", Annals of CIRP, Vol. 29, 1980, pp. 153-156.
5. Gunasekara J.S., *CAD/CAM of Dies*, Ellis Horwood Ltd. Publishers, 1989.
6. Sabaroff et al., "Application of CAD/CAM Technique to Close Tolerance Forging of Spiral Bevel Gears", Annals of CIRP, Vol. 31, No. 1, 1982, pp. 141-144.
7. Sheu J.J. and Lee R.S., "Optimum Die Surface Design of General 3D Section Extrusion by Using a Surface Model with Tension Parameter", Vol. 31, No. 4, 1991, pp. 521-537.
8. Rogers D.F. and Adams J.A., *Mathematical Elements for Computer Graphics*, 2nd Edition, McGraw-Hill, 1990.
9. Mortenson M.E., *Geometric Modelling*, John Wiley & Sons, 1985.
10. Faux I.D and Pratt M.J., *Computational Geometry for Design and Manufacture*, Ellis Horwood, Chichester , 1985.
11. Barnhill R.E," A Survey of the Representation and Design of Surfaces", CG & A, IEEE, October, 1983, pp. 9-16.
12. Vergeest S.M, "Surface Fitting Technique for Interactive Shape Design", Computer in Industry, Vol.13, 1989, pp. 1-13.
13. Vergeest S.M., "Connecting Arbitrary Surfaces under Geometric constraints", Computer in Industry, Vol.8, 1987, pp. 3-12.
14. Choi B. and Lee C.S., "Sweep Surface Modelling via Coordinate Transformation and Blending", CAD, Vol.22., No. 2, March 1990, pp. 87-96.

15. Peigl Les, "On NURBS, A survey", CG & A , IEEE, Jan. 1991, pp. 6-71.

16. Piegl L. and Tiller W., "Curve and Surface Constructions Using Rational B-spline", CAD, vol. 19, No. 9, Nov. 1987, pp. 485-498.

17. Lancaster p. and Salkaskas K., *Curve and Surface Fitting, an Introduction*, Harcourt Brace Javonvich Publishers.

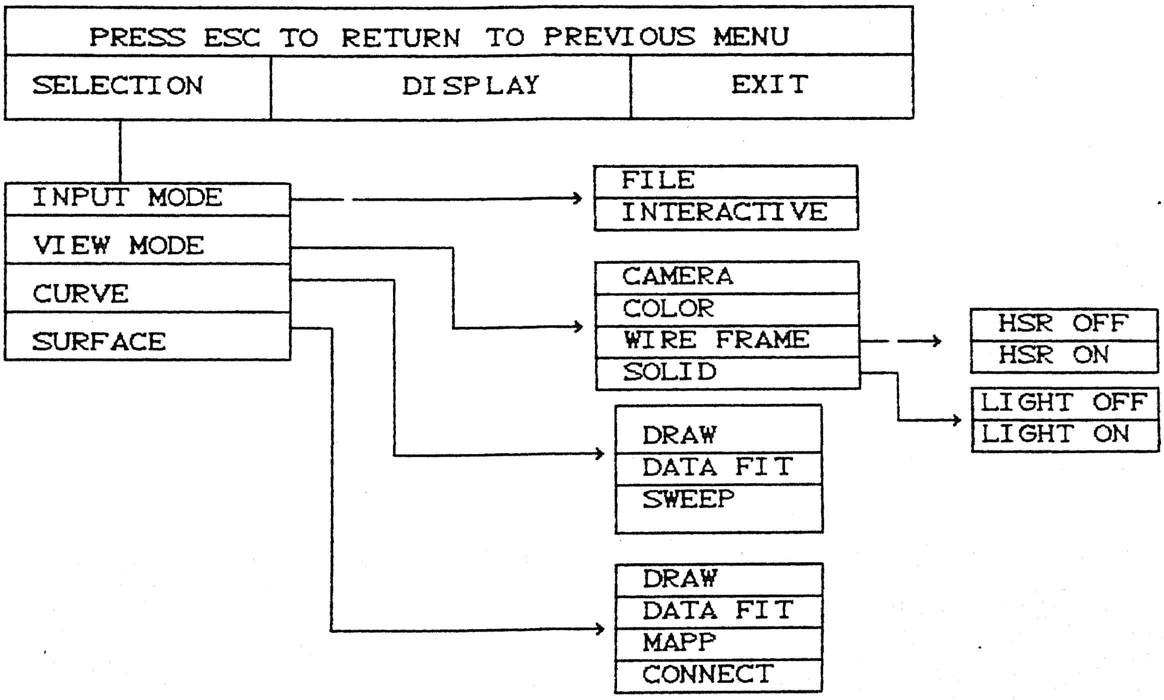
18. Jain M.K. et al., *Numerical Method for Scientific and Engineering Computation*, Wiley Eastern Pbs Ltd.

19. Sabine Coquillart, "Computing Offsets of B-spline Curves", CAD, vol.19, No. 6, 1987, pp. 305-307.

20. Markot R.P. and Magedson R.L, "Procedural Method for Evaluating the Intersection Curves of Two Parametric Surfaces", CAD, vol. 23, No. 6, 1991, pp. 395-404.

APPENDIX

USER´ S MANUAL



ESC This key is to be pressed to return to previous menu.

curser keys are to be used for moving inside the menu.

SELECT: It selects various sub-menu as listed above. Any item can be selected by pressing [return] key. After selection, the necessary value is asked on the right hand corner, for example if input mode and file are selected it asks for file name. If interactive is selected it asks for all values required to make a file. A text file is required in the form as asked by the interactive mode.

DISPLAY: It displays the final graphics. To reach this menu ESC key is to be pressed.

EXIT: When pressed it exits from the main menu. To exit from a text file ^C is to be pressed twice.